No Regret Fiscal Reforms

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Abstract

How should fiscal policy react to shocks *ex-post* while preserving incentives to work and save *ex-ante*? The standard solution involves a commitment to a contingent policy, whereby the initial government sets all the policies for all future states of the world. Contingent policies are unrealistic. As an alternative, I introduce "No Regret Fiscal Reforms": the government has the discretion to change its fiscal policy provided households do not regret their past decisions. Hence flexibility is provided and incentives to work and save are preserved. Such reforms can be achieved by changing taxes on both capital and labor such that wealth effects exactly compensate substitution effects. In a representative agent framework, I study how a benevolent government uses No Regret fiscal reforms and I make comparisons to the optimal contingent policy. Both approaches yield very similar policies and allocations but No Regret reforms entail a small welfare loss. Second, I consider robustness to Near-Rational Expectations i.e the government is uncertain of the households' beliefs about the distribution of shocks and implements a policy robust to this uncertainty. No Regret fiscal reforms are fully robust to this departure from rational expectations. Finally, I characterize No Regret fiscal reforms with wealth and skill heterogeneity.

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1 Introduction

How should fiscal policy react to shocks? Full discretion allows the government to freely adapt its fiscal policy to shocks. The government, however, heavily and repeatedly taxes private wealth because it is a less elastic tax base than labor incomes. Anticipating these taxes, households save little and private wealth is low. Yet, for many reasons, private wealth may be beneficial.[1] A time-inconsistency problem appears: from the perspective of the *current* government it is always optimal to heavily tax private wealth, even though, from the perspective of its *past* self, it would have been optimal not to do so.

The standard solution to encourage private wealth accumulation is to allow the initial government to commit to a fiscal policy that fully binds all future governments. To adapt to shocks, however, the policy must be *contingent* i.e. it must be a function of the realizations of the shocks. But, commitment to a contingent fiscal policy is *technically* unrealistic and not used in practice.

As an alternative, I introduce the "No Regret Fiscal Rule": at each point in time, the government inherits a *status quo* non-contingent fiscal policy and may reform it to adapt to shocks. Any reform, however, must not make households regret their past decisions. These reforms are called "No Regret Fiscal Reforms". The *reformed* fiscal policy then becomes the next-period *status quo* fiscal policy. In other words, the No Regret rule allows for the maximum *current* discretion compatible with *past* promises. Thus, the government has the discretion to adapt its fiscal policy to shocks while being able to reap the benefits associated with private wealth accumulation.

Let us consider a two-period example of a No Regret fiscal reform. At $t = 0$, the government announces a non-contingent policy. At $t = 1$, because of war, for example, the usefulness of public spending relative to private consumption becomes very large. The benevolent government would like to increase taxes to increase public spending. If the capital tax is increased, then households regret having saved so much at $t = 0$. If the labor tax is raised, then households become poorer and regret having saved so little at $t = 0$. It is, therefore, possible to increase (or decrease) both taxes so that wealth effects exactly compensate substitution effects and households do not regret their saving decisions at $t = 0$. This is how No Regret reforms provide flexibility to the government to adapt to shocks at $t = 1$.

[1] Here are some reasons: 1) In an economy with high returns to capital, private capital allows to delay consumption rewards for work efforts, which is less costly. 2) When public spending is exceptionally high, government debt allows the government to pay back over time with smoother tax revenues, which lowers the overall tax distortion. 3) Private savings (or borrowing) allow heterogeneous to hold heterogeneous wealth levels and to match their leisure and consumption needs better (e.g. heterogeneous/stochastic preferences/productivities, life cycle motives). 4) Even though the government could accumulate capital itself, private capital may be better managed in some situations (e.g. because of management agency problems and of lack of market discipline). 5) Households may enjoy wealth for itself.

This paper uses an infinitely-lived representative household model with capital. The benevolent government maximizes households' expected welfare. It taxes linearly labor, capital, and risk-free government bonds to finance public spending. Preferences for public spending are stochastic with some persistence. There are no other shocks. Equilibria under the No Regret rule are such that it is optimal for households to make decisions as if the *status quo* policy would never be reformed because when it is reformed, they will not regret their past decisions. The government's problem, however, is less simple. At each period, the government must find the optimal No Regret fiscal reform, taking into account that the *reformed* non-contingent policy it chooses may itself be reformed in the future. I rewrite this recursive problem using a primal approach i.e. the government directly selects the allocation of resources subject to a resource constraint and an implementability constraint. This implementability constraint is stronger than the one associated with the optimal contingent policy. Thus, lower welfare is achieved under the No Regret rule.

In the absence of shocks, however, these two implementability constraints are identical and both problems yield the same policy, allocation, and welfare. This shows that the No Regret rule addresses time-inconsistency problems as well as full commitment. Although the government has some discretion under the No Regret rule, this discretion does not lead to time-inconsistency issues.

I then rely on numerical simulations to illustrate how the government uses No Regret reforms. First, the government adapts to shocks and spends more when the preference for public spending is high. Second, to finance high public spending, the government *simultaneously* increases the labor tax rate and levies positive taxes on capital and bonds. As explained in the above example, this simultaneity prevents households' regret. Third, since the associated distortion cost is convex, the government keeps the labor tax as smooth as possible. This is made possible thanks to the taxes on capital and bonds that absorb part of the public spending shocks, and thanks to temporary budget deficits financed with public debt. Fourth, when shocks to public spending are low, taxes on capital and bonds are negative. The *ex-ante* wedge associated with these two taxes are very close to zero and saving decisions are hardly distorted.

I then make comparisons with the optimal contingent policy. The main difference is that the labor tax rate is constant, which minimizes distortion costs. The shocks are fully absorbed by taxes (or subsidies) on capital and bonds. These taxes are larger (in absolute values) than the ones under the No Regret rule. These taxes provide full insurance to the government but also lead households to regret their past saving decisions. For this reason, full insurance is not available to a government under the No Regret rule. Optimal No Regret reforms provide important welfare gains over the optimal *non-contingent* policy ($\approx 1.20\%$ in consumption equivalent). The welfare loss compared to the optimal contingent policy is very small ($\approx 0.01\%$ in consumption equivalent).

I also consider robustness to Near-Rational Expectations *à la* Woodford (2010) i.e. the government recognizes that households' beliefs about the distribution of shocks may be different from its own and wants to implement a policy robust to this unknown difference. As it is usual with robust decision making (Hansen & Sargent, 2011), the government's objective is based on the assumption that the worst possible beliefs will always realize. Thus, the government solves a max-min problem. When the uncertainty about households' beliefs is high enough, I show that, when given the freedom to select any contingent policy, the initial government selects a policy equal to the one implemented under the No Regret rule. As a direct consequence, the welfare gap between the optimal contingent policy and the optimal No Regret reforms is fully closed. The intuition for this result is the following. Less insurance may be provided by contingent bonds because, according to the worst beliefs, households believe low returns are more likely and so buy these bonds at low prices. As uncertainty increases, insurance becomes less and less attractive to the government, until it does not resort to insurance. Without insurance, the initial government cannot do better than a government using No Regret fiscal reforms.

Finally, I introduce wealth and skill heterogeneity and prove the existence and provide a partial characterization of No Regret fiscal reforms. Their main characteristic is that households with identical skills but different wealth levels should see their expected utilities shifted by the exact same amount when a No Regret reform is implemented. Thus, the No Regret rule prevents redistribution across households with different wealth levels but with identical skills. In other words, No Regret reforms abide by the Equal Sacrifice principle within each skill group. The Equal Sacrifice principle emerges as a solution to time inconsistency problems, whereas it is usually advocated for equity reasons. Using a mechanism design approach, I show that the No Regret rule provides flexibility to cope with shocks to preferences regarding public spending or regarding redistribution.

Related Literature

This paper studies the optimal financing of a stream of public spending. Several approaches have been considered in the literature.

A widespread approach is to allow the initial government to commit to a contingent policy so that the government can implement the *ex-ante* optimal contingent policy (Chari, Christiano, & Kehoe, 1994; Lucas Jr & Stokey, 1983; Zhu, 1992). Some researchers acknowledged that optimal contingent policies had unrealistic features and made more realistic assumptions. Aiyagari, Marcet, Sargent, and Seppälä (2002) restrict government bonds to be one-period non-contingent bonds. Farhi (2010) adds capital and also restricts capital taxes to be known one period in advance and so cannot be used to cope with shocks. Without capital, Bhandari, Evans, Golosov, and Sargent (2017) allow for more securities and they derive the optimal policies using second-order approximations. In my setup, markets are also incomplete and government bonds and capital taxes cannot provide insurance. Debortoli and Nunes (2010) study loose commitment. They allow governments to renege at each period with a (possibly very small) positive probability. In my setup, the government never reneges.

The opposite approach is full discretion: the government cannot commit at all. Martin (2010) finds that private capital is heavily taxed.[2] In an economy without capital nor contingent bonds, Debortoli, Nunes, and Yared (2017) study the time-inconsistency problem associated with bond prices manipulation. They show that flat maturity structures are used by the government because they mitigate well this time-inconsistency problem. Flat maturity structures, however, fail to provide insurance against shocks. Similarly, the No regret rule addresses time inconsistency problems but fails to provide insurance.

A middle approach is to have limited commitment together with limited discretion. In Klein and Ríos-Rull (2003), the government can commit to the *next period* capital tax and has the discretion to choose the *current* labor tax. I follow this middle approach in my setup: the government commits to abide by the No Regret rule but has some discretion to choose from all the available No Regret reforms.

I contribute to the fiscal part of the "Rule vs Discretion" literature (Kydland & Prescott, 1977). Indeed, contrary to the commitment to a contingent policy, the No Regret rule allows for discretion to adapt to shocks. Close to my work, Athey, Atkeson, and Kehoe (2005) model a present biased benevolent government that uses surplus and deficit to spend a fixed stream of government revenues. As in my paper, the usefulness of public spending is stochastic but my tax revenues are endogenous.[3] They find optimal fiscal rules that restrict the amount the government may spend. Their time inconsistency issue comes from the government's present bias whereas mine comes from the temptation to tax sluggish savings and manipulate bond prices.

I contribute to the large literature on optimal capital taxation. Classic results (Atkinson & Stiglitz, 1976; Chamley, 1986; Judd, 1985) advocate for zero capital taxation, at least in the long run. My model has close to zero *average* capital taxation as in Zhu (1992), Chari et al. (1994), and Kocherlakota (2005).[4]

[2] See also Cohen and Michel (1988)

[3] Halac and Yared (2014) add persistence in the government's taste shocks

[4] Recent research has challenged their results (Straub & Werning, 2020). Apart from this debate, there are many known motives for positive capital taxes or subsidies: precautionary savings under income risk and financial constraints (Aiyagari, 1995), richer agents with different tastes (Saez, 2002), different work elasticities across ages (Erosa & Gervais, 2002), inequality-induced political instability (Farhi, Sleet, Werning, & Yeltekin, 2012), the joy of giving in dynasties, preference for wealth (Saez & Stantcheva, 2018), government more patient than agents (Farhi & Werning, 2007), government caring for future generations (Farhi & Werning, 2010; Piketty & Saez, 2013), agents able to switch their labor income to capital income (Reis, 2011; Smith, Yagan, Zidar, & Zwick, 2019), firms' financial frictions (Abo-Zaid, 2014), imperfect competition in the goods markets (Guo & Lansing, 1999; Judd, 2002), heterogeneity in entrepreneurs' returns (Boar & Knowles, 2018) or in savers' returns (Gerritsen, Jacobs, Rusu, & Spiritus,

My paper follows Savage (1951) and defines regret as the difference between the utility level that could have been obtained had a household known the realized path of the economy in advance and the actual utility level she got. A different definition is introduced by Loomes and Sugden (1982) in their regret theory. They assume that decision-makers choosing among two actions have a utility penalty (boost) called regret (rejoicing) when, for the realized state of the world, the non-chosen action would have yielded a higher (lower) payoff.

My paper introduces robustness to Near-Rational Expectations into fiscal policy. The seminal papers are Woodford (2006) and Woodford (2010), which study monetary policy. Fiscal policies robust to uncertain households' beliefs about shocks have been studied recently by Svec (2012) and Karantounias (2013). They consider the worst households' beliefs *from the households' points of view* whereas I consider the worst households' beliefs *from the government's point of view*. Their governments are facing households with uncertain and pessimistic beliefs, my government is uncertain and pessimistic about households' beliefs. In Svec (2012), the government's objective is to maximize the households' expected welfare computed with *households'* pessimistic beliefs while Karantounias (2013) uses *the government's* beliefs to compute the households' expected welfare. The first has a political government, the second has a paternalistic government. In that sense, my government is paternalistic.

The paper is organized as follows. Section 2 describes the setup. Section 3 introduces the main novelty of this paper, namely No Regret fiscal reforms and the No Regret rule. Section 4 contains numerical illustrations. Section 5 studies Near-Rational Expectations. Section 6 introduces wealth and skill heterogeneity. The last section concludes.

2 The setup

2.1 Environment

The economic environment is a neo-classical growth model with endogenous government expenditures. Time is discrete and the time horizon is infinite with periods denoted t. Preferences regarding public spending are stochastic.[5] They are the only source of uncertainty. These shocks are denoted $\{\theta_t\}_{t=0}^{\infty}$.

Any infinite sequence $\{X_t\}_{t=0}^{\infty}$ will be denoted X and $\{X_s\}_{s=0}^{t}$ will be denoted X^t. When this sequence depends on shocks θ, $\{X_t(\theta^t)\}_{t=0}^{\infty}$ will be denoted $X(\theta)$ and $\{X_s(\theta^s)\}_{s=0}^{t}$

2020). The New Dynamic Public Finance (Golosov, Tsyvinski, Werning, Diamond, & Judd, 2006) also advocates for positive saving distortion. My model carefully avoids all these motives.

[5]Stochastic preferences are a way to conveniently model all shocks that may change the usefulness of public spending although deep preferences are unchanged (e.g. wars)

will be denoted $X^t(\theta^t)$. When no confusion is possible these two sequences will be respectively denoted X and X^t.

Since θ is the only source of uncertainty, I write $\mathbb{E}_t[\ \cdot\ ] \equiv \mathbb{E}[\ \cdot \mid \theta^t\ ]$.[6] For simplicity, θ is a time-homogeneous, irreducible and aperiodic Markov chain that can take N different values and has a non-decreasing transition probability matrix denoted M.[7] At each time t, θ_t is publicly observable and the probability distribution of θ is common knowledge at time $t = 0$. I relax this last assumption later.

Technology. The economy is closed and the production technology is represented by a neo-classical production function $F(\cdot, \cdot)$ with constant returns to scale in capital and labor. Output can be used either for consumption, government expenditures or next-period capital. The resource constraint of the economy at period t when θ^t realized is[8]

$$C_t(\theta^t) + G_t(\theta^t) + K_{t+1}(\theta^t) \leq F(K_t(\theta^t), L_t(\theta^t)) \tag{1}$$

where productive capital is denoted $K_t(\theta^{t-1})$, labor $L_t(\theta^t)$, consumption $C_t(\theta^t)$, government expenditures $G_t(\theta^t)$ and the next period capital $K_{t+1}(\theta^t)$. An *allocation* $\{C, L, G, K\}$ is *resource feasible* when (1) is met at all periods and for all realizations of θ. Perfect competition is assumed so the gross return on capital is equal to the marginal product of capital $F_K(K_t(\theta^t), L_t(\theta^t))$ and the gross wage is equal to the marginal product of labor $F_L(K_t(\theta^t), L_t(\theta^t))$. Perfect competition and constant return to scale in production function imply the absence of profit, so firm ownership is irrelevant.

Households. There are an infinite number of identical households. The households' preferences are given by

$$\mathbb{E}_0\Big[\sum_{t=0}^{\infty} \beta^t\ [\ u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t))\ ]\ \Big]$$

where $C_t(\theta^t)$, $L_t(\theta^t)$ and $G_t(\theta^t)$ are consumption, labor and government expenditures at time t when shocks θ^t realized. $\beta \in (0, 1)$ is the discount factor. I assume that the functions $u(\cdot)$ and $w(\cdot)$ are strictly increasing and strictly concave and the function $v(\cdot)$ is strictly increasing and strictly convex.

The representative household can invest in capital and can buy (or sell) one-period risk-free government bonds. She takes gross wages $F_L(K(\theta), L(\theta))$, gross returns on capital $F_K(K(\theta), L(\theta))$, bond prices $P(\theta)$ and transfers $T(\theta)$ as given. She also takes linear taxes on labor $\tau^L(\theta)$, on capital $\tau^K(\theta)$ and on government bonds $\tau^B(\theta)$ as given.

[6]I use the σ-algebra generated by θ as filtration

[7]i.e. if $\theta' \geq \theta$, then for all x, $\mathbb{P}[\theta_{t+1} \geq x \mid \theta_t = \theta'] \geq \mathbb{P}[\theta_{t+1} \geq x \mid \theta_t = \theta]$

[8]Without loss of generality, F is gross of depreciated capital

I denote net of tax wages $W(\theta) \equiv (1-\tau^L(\theta))F_L(K(\theta),L(\theta))$ and net of tax capital returns $R^K(\theta) \equiv (1-\tau^K(\theta))F_K(K(\theta),L(\theta))$.[9] At period t when the shocks θ^t realized, the households use their net labor incomes $W_t(\theta^t)L_t(\theta^t)$, their non-negative lump-sum transfers $T_t(\theta^t)$, their capital and bonds net returns $R^K_t(\theta^t)K_t(\theta^{t-1})+(1-\tau^B_t(\theta^t))B_t(\theta^{t-1})$ to buy consumption goods $C_t(\theta^t)$, next-period capital $K_{t+1}(\theta^t)$, and bonds $B_{t+1}(\theta^t)$ at price $P_t(\theta^t)$. Thus, the household's budget constraint at period t when the shocks θ^t realized is

$$\begin{aligned} C_t(\theta^t) + K_{t+1}(\theta^t) + P_t(\theta^t)B_{t+1}(\theta^t) \\ = R^K_t(\theta^t)K_t(\theta^{t-1}) + (1-\tau^B_t(\theta^t))B_t(\theta^{t-1}) + W_t(\theta^t)L_t(\theta^t) + T_t(\theta^t) \end{aligned} \tag{2}$$

Households choose consumption, capital and bonds to maximize their expected utility subject to their budget constraints (2). Optimal saving using capital investment at period t gives the following Euler Equation

$$u'(C_t(\theta^t)) = \mathbb{E}_t[\ \beta R^K_{t+1}(\theta^{t+1})u'(C_{t+1}(\theta^{t+1}))\ ] \tag{3}$$

Optimal saving using bonds at period t gives another Euler Equation

$$u'(C_t(\theta^t)) = \mathbb{E}_t[\ \beta R^B_{t+1}(\theta^{t+1})u'(C_{t+1}(\theta^{t+1}))\ ] \tag{4}$$

where $R^B_{t+1}(\theta^{t+1}) \equiv (1-\tau^B_{t+1}(\theta^{t+1}))/P_t(\theta^t)$ is the the net of tax return on bonds. Optimal labor effort any period t implies

$$v'(L_t(\theta^t)) = W_t(\theta^t)u'(C_t(\theta^t)) \tag{5}$$

Proofs are in the Appendix.

Government. The government is benevolent so its preferences are identical to the households'. Government directly sets its expenditures.[10]

To fund its stream of expenditures $G(\theta)$ and positive lump-sum transfers $T(\theta)$, the government raises linear taxes on labor incomes $\tau^L(\theta)$, on the households' capital assets (including returns) $\tau^K(\theta)$, and on maturing bonds $\tau^B(\theta)$. At each period t, the government may also sell a quantity $B_{t+1}(\theta^t)$ of one-period bonds at price $P_t(\theta^t)$ and must pay back the maturing bonds $B_t(\theta^{t-1})$. When $B_{t+1}(\theta^t)$ is negative the government holds claims on households. Therefore the government's budget constraint at each time t is

$$G_t(\theta^t) + T_t(\theta^t) + (1-\tau^B_t(\theta^t))B_t(\theta^{t-1}) \tag{6}$$

[9] Capital *and* capital incomes are taxed at rate τ^K.
A tax with a rate $\frac{\tau^K F_K(K(\theta),L(\theta))}{F_K(K(\theta),L(\theta))-1}$ applied to capital incomes *only* is exactly equivalent.

[10] The usual assumption is that public expenditures are exogenous but endogenous public expenditures are more realistic. Furthermore, endogenous public expenditures imply additional time inconsistency problems, which my approach also addresses well. See Debortoli and Nunes (2013) who gets significantly different results than Krusell, Martin, Ríos-Rull, et al. (2006) by assuming endogenous spending

$$= \tau_t^L(\theta^t) F_L(K_t(\theta^{t-1}), L_t(\theta^t)) L_t(\theta^t) + \tau_t^K(\theta^t) F_K(K_t(\theta^{t-1}), L_t(\theta^t)) K_t(\theta^{t-1}) + B_{t+1}(\theta^t) P_t(\theta^t)$$

I also impose natural debt limits both to the government and to households, which rule out Ponzi schemes.

Competitive Equilibrium. I may now define my equilibrium concept:

Definition 1. ***(Competitive Equilibrium)***
A competitive equilibrium is a contingent allocation $\{C_t, L_t, G_t, K_{t+1}\}_{t=0}^{\infty}$, a contingent policy $\{G_t, T_t, \tau_t^L, \tau_t^K, \tau_t^B, B_{t+1}\}_{t=0}^{\infty}$ and contingent prices $\{W_t, R_t^K, P_t\}_{t=0}^{\infty}$ such that:

1. *Firms maximize their profits, which implies $W_t = (1-\tau_t^L) F_L(K_t, L_t)$ and $R_t^K = (1-\tau_t^K) F_K(K_t, L_t)$*

2. *Household's decisions $\{C_t, L_t, K_{t+1}, B_{t+1}\}_{t=0}^{\infty}$ are optimal, which implies equations (3),(4) and (5).*

3. *The resources constraint (1) is always met.*[11]

Such allocations and policies are called *implementable*. When households' decisions are optimal *under* a policy, I say that this policy *induces* these households' decisions. Any implementable allocation can be implemented with a commitment to a contingent policy at $t = 0$. This commitment ability is sufficient but not necessary, typically the optimal policy under full discretion is implementable without any commitment ability. Note that households are rational i.e. they know from $t = 0$ the policy that will realize for each realization of the shocks.[12] In other words, there is no "surprise" policy.

In the next section, I introduce a constraint on fiscal policy, namely the No Regret rule, so that only a subset of these implementable allocations will be available to the government.

3 No Regret Fiscal Reforms

When the initial government commits to a contingent policy, it fully binds all future governments. The opposite approach is full discretion, no government may bind any of its successors. This paper's approach is halfway between these two extremes: each government may *partially bind* future governments so that each government has *some discretion* left. It works as follows: the initial government chooses an implementable *non-contingent* policy and each successive government may reform this policy subject to the No Regret rule. These reforms are called No Regret fiscal reforms. Let me now formally define the No Regret rule.

[11] Since the economy is closed, the resource constraint (1), and the households' budget constraint (2) imply by Walras' law that the government's budget constraint (6) holds as well.

[12] Even when it is the result of a complicated game between households and the government, like the one under full discretion

3.1 No Regret rule

Let me first define a *fiscal reform*.

Definition 2. ***(Fiscal Reform)***
A fiscal reform at time t_R is a change of policy, so that the status quo policy $\{G_s, T_s, \tau_s^L, \tau_s^K, \tau_s^B, B_s\}_{\{s \geq t_R\}}$ is replaced by a new policy $\{\tilde{G}_s, \tilde{T}_s, \tilde{\tau}_s^L, \tilde{\tau}_s^K, \tilde{\tau}_s^B, \tilde{B}_s\}_{\{s \geq t_R\}}$

I call *current policy* the policy chosen by the government i.e. the *status quo* policy if there is no reform, or the new policy if there is a reform. At the beginning of the next period, the government inherits this *current* policy as its *status quo* policy. At each period t, the timing is the following. First, θ_t is observed. Then a fiscal reform is announced if the government wishes to. Finally, households take their decisions. In this paper, I impose that fiscal reforms are subject to the No Regret rule, which I now formally define.

Definition 3. ***(No Regret rule)***
Under the No Regret rule, a fiscal reform at time t_R must be such that:

1. *The new policy is implementable and non-contingent*

2. *The optimal households' decisions before t_R are identical*

    - *Under the status quo policy*
    - *Under the new policy*

Fiscal reforms that respect the No Regret rule are called No Regret fiscal reforms. Note that households are rational and that they know that the current policy may be reformed, so, what decisions do they take? The next lemma establishes that, even though the current policy may be reformed, under the No Regret rule, it is optimal for households to make decisions as if it were common knowledge among them that the current policy would never be reformed.

Lemma 1. ***(Equilibrium decisions under the No Regret rule)***
Under the No Regret rule, households' decisions are induced by the current policy.

Thanks to the No Regret rule, taking decisions induced by current policies allows households' *past* decisions to always be optimal under the current policy. So, whatever the reforms announced by the government, the past decisions are the best decisions that any atomistic household could have taken. If they are *ex-post* optimal then they also are *ex-ante* optimal. In other words, the best strategy for households is to make decisions as if the current policy is never reformed and to (rightfully) trust the government to never make them regret this strategy.

In this setup, shocks are first affecting the government's preferences, and then the government's reforms in reaction to these shocks affect households. So households are only *indirectly* affected by shocks. According to the previous lemma, households do not have to take into account future reforms when making decisions. Thus, they do not have to take into account future shocks. This implies that shocks could be privately observed by the government as in Sleet (2004), this would affect neither the government's policy nor the households' decisions. Furthermore, the households' beliefs about the distribution of shocks could be different from the government's, the same result would hold.

Of course, other types of shocks could *directly* affect households. Then households should take them into account when making decisions. This suggests that, under the No Regret rule, households would only have to take into account the shocks that do not affect the government policy. A typical example would be idiosyncratic shocks.

No Regret. Let us now discuss the terms "No Regret". The notion of "Regret" has been introduced in the theory of decision under uncertainty by Savage (1951). A decision maker chooses between several actions. Each action yields different payoffs depending on the state of the world. For each state of the world, there is the best *ex-post* action. The *regret* associated to an action and to a state of the world is the utility gain that could have been achieved if the best *ex-post* action had been chosen *ex-ante*. In my setup, the actions are the households' decisions, the state of the world is the realized shock and the policy chosen by the government. Regret is the maximum gain expressed in *total utility* under the new policy, which could have been achieved with other past decisions compared to the *actual* decisions that were taken before the reform.[13]

The No Regret rule imposes that optimal households' decisions under the *status quo* policy are optimal under any other *new* policy. As a consequence, no utility gain could have been achieved with different past households' decisions. Thus, households have no regret.

Informed households. Under the No Regret rule, when a No Regret reform is implemented, households do not regret their past decisions. This implies that, had they known about the reform in advance, they wouldn't have behaved differently.

3.2 Ex Post Euler Equation

Under the No Regret rule, whether there are reforms or not, households do not regret their past decisions. In other words, *ex-ante* optimal households' decisions are always *ex-post* optimal. As a consequence, at any time t and for any realization θ^t, households

[13]Total utility means the utility is computed from $t = 0$. If utility were computed from the time of the reform, households would always regret that their past selves had saved much more since past work efforts are not taken into account.

do not regret their saving decisions at time $t-1$ i.e. their capital investment $K_t(\theta^{t-1})$ and the number of bonds they bought $B_t(\theta^{t-1})$. This implies that capital and bonds always have the same net returns denoted R. If one of these two net returns were higher, households would regret not having arbitraged the difference. Furthermore, the following equation, called the *Ex Post Euler Equation*, always holds.

Lemma 2. ***(Ex Post Euler Equation)***
Under the No Regret rule, the following Ex Post Euler Equation holds for any realization θ^t.

$$u'(C_{t-1}(\theta^{t-1})) = \beta R_t(\theta^t) u'(C_t(\theta^t)) \tag{7}$$

To see this, assume that the left-hand-side term is smaller (larger). Then, households could have got higher total welfare with higher (lower) savings at time $t-1$. As a consequence, they regret their saving decisions at time $t-1$. This is impossible under the No Regret rule.

Four things are worth mentioning. First, the Ex Post Euler Equation (7) implies the two Euler equations for capital (3) and bonds (4). Second, since $R^B = R^K$, households are indifferent between government bonds and capital. Thus, the compositions of their portfolios are indeterminate and only their total wealth matters. Third, since capital and bonds have the same ex post net returns it is useless for the government to trade capital on top of its own bonds. Owning and trading capital won't make the markets less incomplete nor provide any insurance to the government, contrary to the case with *untaxed risk-free* bonds studied by Farhi (2010). Fourth, under the No Regret rule, considering only one-period bonds instead of having a full maturity structure is without loss of generality since all assets should yield the same net of tax returns.

Risk-free bonds. Both Aiyagari et al. (2002) and Farhi (2010) used *untaxed risk-free* bonds whereas, in my model, risk-free bonds are taxed linearly to guarantee the absence of regret. Their bonds are risk-free in the sense that their returns are non-contingent. My bonds are risk-free in the sense that households value its net of tax payoffs similarly in each state of the world.[14] In both cases, bonds cannot provide insurance against shocks to public spending.

"Risk-free" capital taxes. A similar parallel can be drawn with the capital tax. Farhi (2010) restricts capital taxes to be known one period in advance (i.e. they are "risk-free" when capital is invested). My capital taxes must be such that households value net returns similarly in each state of the world. In both cases, capital taxes cannot provide insurance against shocks to public spending, as it was the case with risk-free bonds.

[14]With decreasing marginal utility of consumption, net returns are lower in states of the world in which households consume less.

3.3 Relaxed government problem

At each time t, the government's problem is to find the optimal No Regret reform. This reform may change current and future prices and so will *indirectly* induce households to make decisions $C_t, L_t, K_{t+1}, B_{t+1}$. In this subsection, I study a relaxed problem where the government *directly* chooses current prices R_t, W_t, P_t and current households' decisions $C_t, L_t, K_{t+1}, B_{t+1}$. This *direct* choice is subject to four constraints that always hold under the No Regret rule: the resource constraint (1), the households budget constraint (2), the optimal labor equation (5), and the ex post Euler Equation (7). At each time t, the government maximizes its utility $V_t(\cdot)$, which is the sum of its *current* flow utility and of all the (discounted) flow utilities of future governments. So, its utility $V_t(\cdot)$ is the sum of its *current* flow utility and of the (discounted) utility of the next period government $\beta V_{t+1}(\cdot)$. Hence the problem has the following recursive form

$$V_t(C_{t-1}, K_t, B_t, \theta_t) = \max_{\substack{C_t, L_t, G_t, K_{t+1} \\ R_t, W_t, B_{t+1}, T_t}} u(C_t) - v(L_t) + \theta_t w(G_t)$$
$$+ \beta \mathbb{E}_t[\, V_{t+1}(C_t, L_t, K_{t+1}, B_{t+1}, \theta_{t+1}) \,]$$
$$\text{s.t.} \quad (1), (2), (5), (7)$$

This relaxed problem has three endogenous state variables and eight control variables. It can be rewritten with two endogenous state variables and five control variables. To do so, I define the households' total net asset position expressed in marginal utility

$$A_{t+1} \equiv u'(C_t)(K_{t+1} + P_t B_{t+1})/\beta$$

and I use the stationarity of the relaxed problem, and the fact that transfers are positive $T_t \geq 0$ to rewrite the relaxed problem as follows

$$V(K_t, A_t, \theta_t) = \max_{\substack{C_t, L_t, G_t \\ K_{t+1}, A_{t+1}}} u(C_t) - v(L_t) + \theta_t w(G_t) \; + \; \beta \mathbb{E}_t[\, V(K_{t+1}, A_{t+1}, \theta_{t+1}) \,] \tag{8}$$
$$\text{s.t.} \quad C_t + G_t + K_{t+1} \leq F(K_t, L_t) \qquad (\lambda_t)$$
$$\text{to} \quad A_t \leq u'(C_t)C_t - v'(L_t)L_t + \beta A_{t+1} \qquad (\gamma_t)$$

The first inequality is the resource constraint and the second is called the implementabilty constraint. The associated Lagrange multipliers are λ_t and γ_t respectively.

The initial government's problem. At time $t = 0$, the initial government may choose any implementable non-contingent policy. As it is well known, the government would like to heavily tax accumulated wealth to finance its future expenditures without having to distort the economy. A 100% tax on capital and bonds yields $A_0 = 0$. To model a limit to initial wealth taxation, the initial households' wealth A_0 is exogenously fixed.

Convergence to First Best. When $\gamma_t = 0$, the implementability constraint is slack and only the resource constraint binds i.e. it is the First Best problem. In other words, the government is wealthy enough to pay for the First Best expenditures without any distortive taxation. When $\gamma_t > 0$, the government must tax households in order to finance its expenditures or its debt. The higher the usefulness of expenditures or the higher its debt, the higher the multiplier γ_t. Furthermore, the government chooses to accumulate assets to self insure against shocks so that, in the long run, it becomes rich enough to self-finance the First Best expenditures. Distortive taxation is then no longer needed, the economy has reached First Best.

Lemma 3. ***(Convergence to First Best)***
Under the No Regret Rule, it is almost sure that the economy eventually reaches First Best.

The proof is in the Appendix.

Comparison with commitment to a contingent policy. It can been shown that the problem of the initial government committing to a contingent policy solves the following recursive problem at each time t.

$$\tilde{V}(K_t, A_t, \theta_t) = \max_{\substack{C_t, L_t, G_t \\ K_{t+1}, A_{t+1}(\cdot)}} \quad u(C_t) - v(L_t) + \theta_t w(G_t) \ + \ \beta \mathbb{E}_t[\, \tilde{V}(K_{t+1}, A_{t+1}(\theta_{t+1}), \theta_{t+1})\,] \tag{9}$$

$$\text{s.t.} \quad C_t + G_t + K_{t+1} \leq F(K_t, L_t) \qquad (\lambda_t)$$

$$\text{to} \quad A_t \leq u'(C_t)C_t - v'(L_t)L_t + \beta \mathbb{E}_t[\, A_{t+1}(\theta_{t+1})\,] \qquad (\gamma_t)$$

where the next period households' wealth $A_{t+1}(\cdot)$ is contingent on the realization of the next period shock θ_{t+1}. Problems (8) and (9) are very close. The difference lies in the fact that, for the problem (9), the implementability inequality must be met in expectation, whereas, for the problem (8), it should hold for all realizations of the shocks θ. The fact that households' wealth $A_{t+1}(\cdot)$ can vary across realization of shocks is made possible by contingent bonds or contingent capital taxes. They allow the government to be richer for high realizations of shocks θ_{t+1} and so to smooth tax distortion across realizations of shocks. This also implies that, when there is no shock, both problems have the same solution. Under the No Regret rule, the initial government may announce a non-contingent policy which is not reformed in the absence of shocks. This means that the No Regret rule prevents time inconsistency problems such as heavy capital taxation, default on government debt, bonds' prices manipulation with labor taxes and government spending.

3.4 Sequential problems

The previous recursive problems can be rewritten in sequential forms. The value function $V(\cdot,\cdot,\cdot)$, solution to the recursive problem, solves the following *sequential* problem:

$$V(K_0, A_0, \theta_0) = \max_{\substack{\{C_t(\theta^t), L_t(\theta^t), \\ G_t(\theta^t), K_{t+1}(\theta^t)\}_{t\geq 0}}} \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \, [\, u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t)) \,]\,] \tag{10}$$

$$\text{s.t. } C_t(\theta^t) + G_t(\theta^t) + K_{t+1}(\theta^t) \leq F(K_t(\theta^{t-1}), L_t(\theta^t)) \text{ for all } \theta$$

$$\text{to } A_0 \leq \sum_{t=0}^{\infty} \beta^t \, [\, u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t) \,] \text{ for all } \theta$$

Similarly, the value function $\tilde{V}(\cdot, \cdot, \cdot)$, solution to the recursive problem of the government that commits to a contingent policy, solves the following *sequential* problem:

$$\tilde{V}(K_0, A_0, \theta_0) = \max_{\substack{\{C_t(\theta^t), L_t(\theta^t), \\ G_t(\theta^t), K_{t+1}(\theta^t)\}_{t\geq 0}}} \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \, [\, u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t)) \,]\,] \tag{11}$$

$$\text{s.t. } C_t(\theta^t) + G_t(\theta^t) + K_{t+1}(\theta^t) \leq F(K_t(\theta^{t-1}), L_t(\theta^t)) \text{ for all } \theta$$

$$\text{to } A_0 \leq \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \, [\, u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t) \,]\,]$$

These two problems are similar. The difference lies in the fact that for the second problem the implementability inequality must be met in expectation whereas for the first it should hold for all realizations of the shocks θ. This directly implies that the optimal contingent policy achieves a higher welfare than the optimal No Regret policy. Numerical simulations later show that this difference is very small.

In the absence of shocks, both implementability constraints are identical and both problems yield the same policy, allocation, and welfare. This shows that the No Regret rule addresses time-inconsistency problems as well as full commitment. Although the government has some discretion under the No Regret rule, this discretion does not lead to time-inconsistency issues.

3.5 Tax implementation with simple No Regret reforms

Tax Indeterminacy. At every time t, the government may implement a new policy $\{G_s, T_s, \tau_s^L, \tau_s^K, \tau_s^B, B_s\}_{\{s\geq t\}}$ to induce households to take the desired decisions C_t,L_t. There are many such policies because the number of degrees is infinite. This is a situation of *tax indeterminacy*. To address this problem I propose to restrict the number of degrees of freedom by introducing *simple* No Regret reforms.

3.5.1 Simple No Regret reforms

As discussed above, at each reform time t_R, there are many different No Regret reforms that induce households to take the same decisions at time t_R. I find it useful to consider a subset of reforms called *simple* reforms. They are simple for several reasons. First, the labor tax is constant. Second, transfers and taxes on capital and bonds are *one-off* i.e. they are always null except at the time of the reform t_R. Third, government spending is constant after time $t_R + 2$.

Definition 4. ***(Simple fiscal reform)***
A fiscal reform at time t_R is called simple when the new non-contingent policy $\{G_s, T_s, \tau^L_s, \tau^K_s, \tau^B_s, B_{s+1}\}_{s=t_R}^{\infty}$ is such that:

- *Taxes on labor are constant i.e. $\tau^L_s = \tau^L_{t_R}$ for any time $s \geq t_R + 1$*
- *Taxes on capital and bonds are one-off i.e. $\tau^K_s = \tau^B_s = 0$ for any time $s \geq t_R + 1$*
- *Transfers are one-off i.e. $T_s = 0$ for any time $s \geq t_R + 1$*
- *Government spending is constant after time $t_R + 2$*
  i.e. $G_s = G_{t_R+2}$ for any time $s \geq t_R + 2$

Tax implementation. For given past consumption C_{t_R-1}, capital K_{t_R} and bonds B_{t_R}, let us study the implementation of the desired allocation $C_{t_R}, L_{t_R}, G_{t_R}$ using a simple No Regret reform.

Optimal labor effort at time t_R (equation (5)) pins down the constant labor tax

$$\tau^L_{t_R} \equiv 1 - \frac{v'(L_{t_R})}{F_L(K_{t_R}, L_{t_R})u'(L_{t_R})}$$

The Ex Post Euler Equation pins down taxes on capital and bonds

$$\tau^K_{t_R} \equiv 1 - \frac{u'(C_{t_R-1})}{\beta F_K(K_{t_R}, L_{t_R})u'(C_{t_R})}$$
$$\tau^B_{t_R} \equiv 1 - \frac{u'(C_{t_R-1})P_{t_R}}{\beta u'(C_{t_R})}$$

The current government spending G_{t_R} is directly set as desired by the government. The path of future government spending is defined by G_{t_R+1} and $G_{s \geq t_R+2}$. G_{t_R+1} can be chosen freely but then $G_{s \geq t_R+2}$ is constrained by the government's budget. By choosing G_{t_R+1}, the government decides how front-loaded or back-loaded the path of future government spending is. More Front-loading (back-loading) implies more (less) government bonds and higher (lower) returns on wealth R_{t_R+2}. Accordingly, households decide to consume less (more), to work more (less) and to invest more (less) today. So a well chosen front-loading of future government spending induces households to consume the desired amount C_{t_R}. The next lemma summarizes these results.

Lemma 4. ***(Simple No Regret reforms)***
Any allocation implementable under the No Regret rule can also be implemented by uniquely defined simple No Regret fiscal reforms.

Using only simple No Regret reforms implies that, at every time t, the government has six policy instruments. The *change* in the constant labor tax, the two one-off taxes (or subsidies) on capital and on bonds, and the government expenditures for the current period t and the next two periods. When the government wishes to reform the *status quo* policy, it uses these instruments so that the government's budget constraint and the No Regret rule are respected. Let us discuss them in turn.

Change in labor tax. The wealth effect of a labor tax hike makes households consume less, which increases marginal utility. According to the Ex Post Euler equation (7), households wish they had saved more before the reform. A labor tax hike increases tax revenues. The effect on labor supply is ambiguous and depends on parameters.

One-off taxes on capital and on bonds. One-off taxes on capital and on bonds decrease net return on savings. Their wealth effects make households consume less and work more. With usual parameters, their price effect is larger than their wealth effect, thus households wish they had saved less before the reform to meet the Ex Post Euler equation (7). One-off taxes on capital and on bonds have to be set jointly to guarantee that bonds and capital have the same net returns to avoid regret. These taxes bring tax revenues to the government's budget.

Government expenditures. Higher government expenditures must be met by higher taxes. However, it is possible to change the path of government expenditures without changing taxes. Roughly speaking, expenditures can be front-loaded or back-loaded. For example, more front-loaded expenditures are financed with additional government bond emissions. These emissions crowd out capital and increase net returns in the short term. As a consequence, households back-load consumption and front-load labor supply. This has two effects on the Ex Post Euler equation (7). First, less *current* consumption increases marginal utility, second more *current* labor supply increases the marginal return of capital. According to the Ex Post Euler equation (7), households wish they had saved more before the reform.

4 Simulations

4.1 Specifications

In this section, I describe numerical simulations of the optimal policy under the No Regret rule. Let us first describe the specifications I use.

Technology and stochastic preferences. I use a Cobb-Douglas production function with capital share α and with a depreciation rate of capital δ.

$$F(K, L) = K^{\alpha} L^{1-\alpha} + (1 - \delta)K$$

I use iso-elastic preferences regarding consumption, labor and government spending

$$u(C) - v(L) + \theta w(G) = \frac{C^{1-\sigma} - 1}{1 - \sigma} - \kappa \frac{L^{1+\chi} - 1}{1 + \chi} + \theta \frac{G^{1-\xi} - 1}{1 - \xi}$$

The shock process θ is modeled as a two state Markov chain with a constant transition matrix. The two states are denoted H and L (High and Low). The transition matrix is

TRANSITION MATRIX

$\mathbb{P}[\ \theta_{t+1} = H \mid \theta_t = H\ ]\quad = p_{HH}$	$\mathbb{P}[\ \theta_{t+1} = L \mid \theta_t = H\ ]\quad = 1 - p_{HH}$
$\mathbb{P}[\ \theta_{t+1} = L \mid \theta_t = L\ ]\quad = p_{LL}$	$\mathbb{P}[\ \theta_{t+1} = H \mid \theta_t = L\ ]\quad = 1 - p_{LL}$

4.2 Parameters

I use a yearly time period as it is standard in fiscal policy. The parameters are in table 1. The discount factor β is 0.96, the intensive Frisch elasticity $1/\chi$ is 0.5, the private relative risk aversion σ is 1, which are standard values. $\sigma = 1$ corresponds to the log utility which allows having balanced wealth and substitution effects, which simplifies the exposition. I choose to set the public relative risk aversion ξ to 1, to be equal to the private one. The capital share α in the Cobb-Douglas production function is 0.33. I set p_{HH} to $3/4$ and p_{LL} to $5/6$ so that the average length of high and low shocks are 4 and 6 years respectively. This implies that, on average, the economy is in the low state 60% of the time. The deprecation rate δ, the disutility of labor parameter κ and the utility of government spending average parameter $\mathbb{E}[\theta_t]$ are respectively calibrated at 6.83%, 0.72 and 0.1 in order to have a capital to output ratio equal to 3, labor normalized to 1 and a government spending share of the output of 20% in a steady-state, unshocked economy. The standard deviation of the shocks θ is set so that the standard deviation of the government spending is 5% of GDP.

EXOGENOUSLY CHOSEN AND CALIBRATED PARAMETERS		
Discount factor β	0.96	-
Intensive Frisch elasticity $1/\chi$	0.5	Chetty, Guren, Manoli, and Weber (2011)
Private relative risk aversion σ	1	Calvet, Campbell, Gomes, and Sodini (2021)
Public relative risk aversion ξ	1	Identical to σ
Capital share α	0.33	-
p_{HH}	$3/4$	4 year expected length for High
p_{LL}	$5/6$	6 year expected length for Low
δ	6.83 %	K/Y=3
κ	0.72	Normalization labor 1
$\mathbb{E}[\theta_t]$	0.10	Gov spending = 20 % of GDP
$\sigma[\theta_t]$	0.16	σ(Gov spending) = 5 % of GDP

Table 1: Parameters

4.3 Optimal policy

Let me now discuss the optimal allocation and the optimal policy implemented by a government under the No Regret rule. In the following simulations, I assume that the government only uses simple No Regret reforms.

Sample path. I show the realized allocation and policy for one sample path which is one realization of the shock process θ. The economy starts with the capital of the *unshocked* steady state. Then, for 6 periods, shocks are low i.e. $\theta_t = L$, which is the expected length of low shocks. Then the shocks are high for 4 periods (grey zone), which is the expected length of high shocks. Finally, shocks are low until the end of time. Note that, at any time, the government does not know the future shocks.

Government expenditures. The most important thing to note is that the government spends more when preferences for public spending are high, as is displayed in Figure 1. Under the No Regret rule, the government has enough discretion to spend more when it is needed.

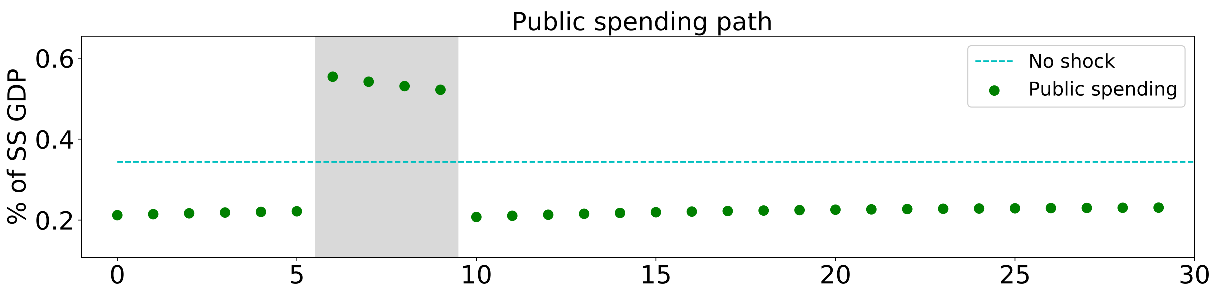

Figure 1: Public spending

labor tax. Of course, the government needs to finance its expenditures. The main tool used by the government to raise tax revenues is the labor tax. Shocks are persistent, so when the shock θ goes from low to high, the government learns that *expected* public spending will be high in the near future. The welfare cost of the labor tax distortion is convex in the labor tax rate. Thus, to smooth the labor tax rate, the government anticipates these high future spending and raises the labor tax right away. Thus, as displayed in Figure 2, the government increases the tax rate on labor when the shock goes from low to high. Note that the labor tax also increases when the shock *stays* high but less dramatically. The size of the change in labor tax rate is related to the *size of the news*: because shocks are persistent, tax changes are more dramatic when the shock goes from low to high or from high to low compared to when it stays high or stays low.

According to lemma (1), households make decisions as if the current policy would never be reformed. And the government uses only simple No Regret reforms. So, changes in the current labor tax also affect the expected net labor incomes of all future periods. This amplifies the income effect that labor tax changes have on current consumption. When the shock goes from low to high, the labor tax increases and makes households poorer, as a consequence they regret having saved so little in the past.

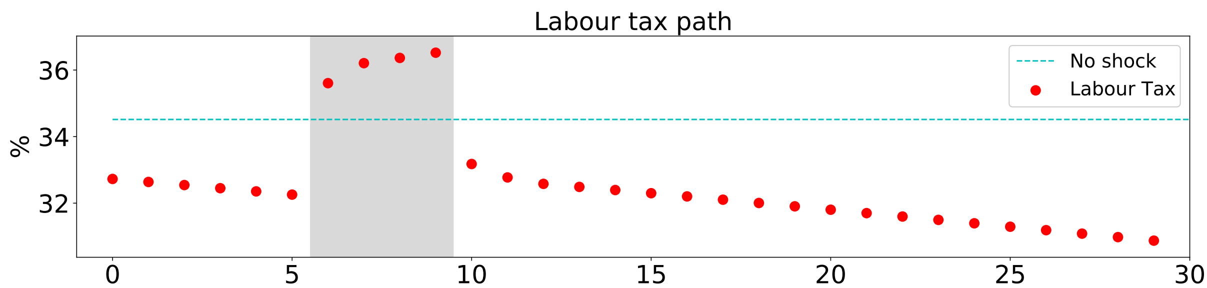

Figure 2: labor Tax

One-off taxes on capital and bonds. As discussed above, labor tax increases make households regret having saved so little. Positive one-off taxes on capital and on bonds have the opposite effect. To guarantee the absence of regret, the government sets positive one-off taxes on capital and on bonds so that the Ex-Post Euler Equation (7) holds.

These additional tax revenues lessen the need to increase the labor tax. As shown in Figure 3, the sizes of one-off capital tax (or subsidy) are related to the *size of the news*. The tax on bonds follows a similar path.

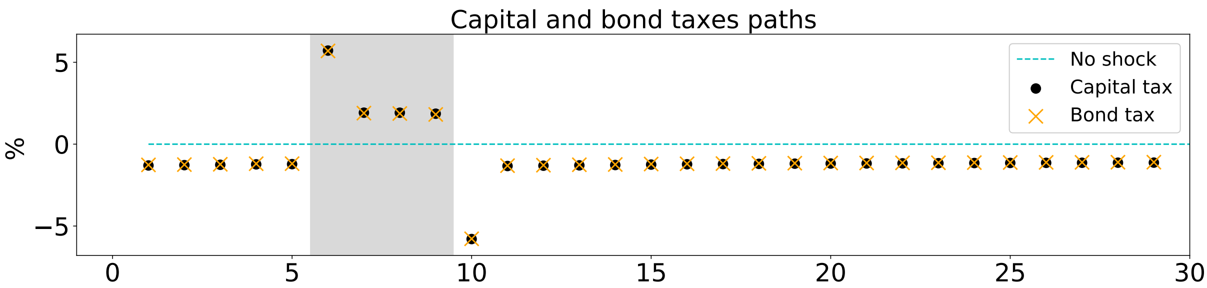

Figure 3: Capital and bond taxes

Consumption and labor. labor tax changes and one-off taxes on capital and bonds have effects on current consumption and labor. Changes in the labor tax change current and expected net wages. One-off taxes (or subsidies) on capital bonds modify accumulated savings. When the shock goes from low to high, the labor tax increase and one-off taxes on capital and bonds are positive. Both induce consumption to jump down because of income and wealth effects. Symmetrically, the consumption jumps up when the shock goes from high to low.

When the shock *stays* high, three things happen. First, taxes on labor income become higher. Second, the government debt increases (see Figure 5). Third, the capital stock decreases (see Figure 6). This increases expected gross returns in the near future. Furthermore, because capital and labor are complement, this decreases expected gross wages in the near future. The capital effect is the strongest and drives the decrease of consumption and the increase in labor supply. Symmetrically, when the shock *stays* low, the capital effect drives the increase of consumption and the decrease in labor supply. When the capital stock reaches a plateau, so do consumption and labor.

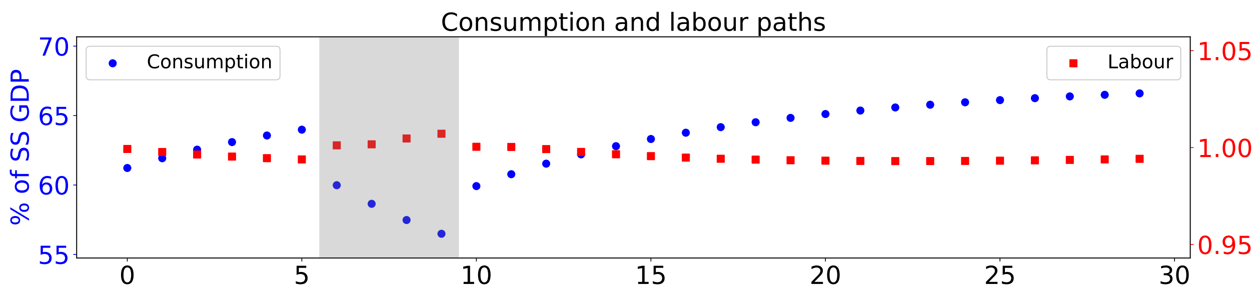

Figure 4: Consumption and labor

Public debt. The path of the public debt is displayed in Figure 5. As it is usual, government debt is used to smooth tax distortion across time periods. So when shocks keep being low, the public debt decreases and when shocks keep being high, the public debt increases. When the shock goes from low to high, large one-off taxes on capital and bonds allows the government to pay back a large fraction of its debt. Symmetrically, when the shock goes from high to low, the government makes important debt emissions to finance the large one-off subsidies on capital and bonds. One important consequence is that these one-off taxes/subsidies play an insurance role. Indeed, public debt is low (high) when public spending is high (low).

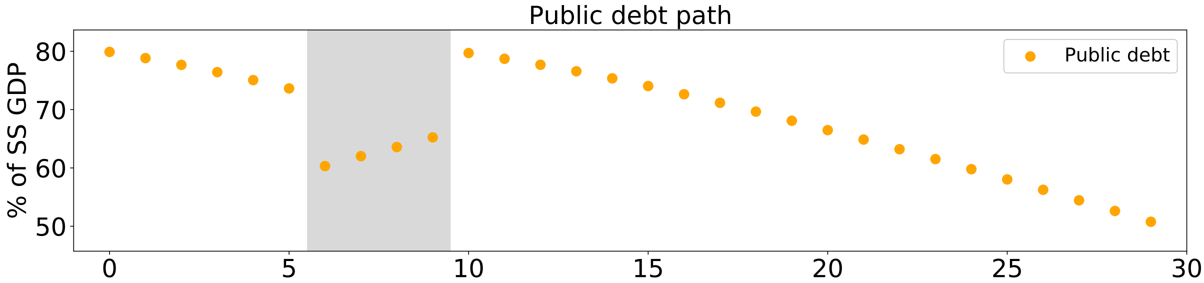

Figure 5: Public debt

Capital. As displayed in Figure 6, the capital stock decreases when shocks are high and increases when shocks are low. In other words, capital is used as a buffer to cope with spending shocks. However, capital depreciates (at rate δ) and marginal returns to capital are decreasing. Thus, maintaining a high buffer stock of capital is costly. For this reason, there is a maximum capital buffer stock towards which the capital stock converges when shocks keep being low. Symmetrically, a low level of capital implies high returns on capital, which are costly to let go. As a consequence, there is a lower limit to the capital stock towards which the economy converges when shocks keep being high (not shown in these simulations). Note that when capital is higher, the economy is richer so consumption and expenditures are higher and labor efforts are lower (see Figure 1 and Figure 4).

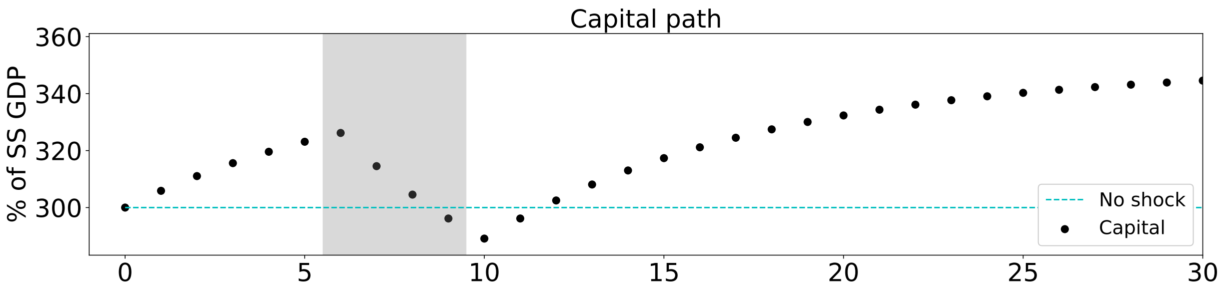

Figure 6: Capital

Realized and planned public spending. As we have seen, the capital stock is used a buffer. When public spending is high, the capital stock decreases. When public spending is low, the capital stock increases to prepare for high shocks. But how does the government induce the capital stock to increase or to decrease? With simple reforms, at each time t, the government sets 3 points of the public spending path. The *current* spending G_t, the *next-period* spending G_{t+1} and the *ever-after* spending $G_{s\geq t+2}$. They are displayed in Figure 7. Of course, the public spending path is constrained by the government budget, which pins down the *ever-after* spending $G_{s\geq t+2}$.

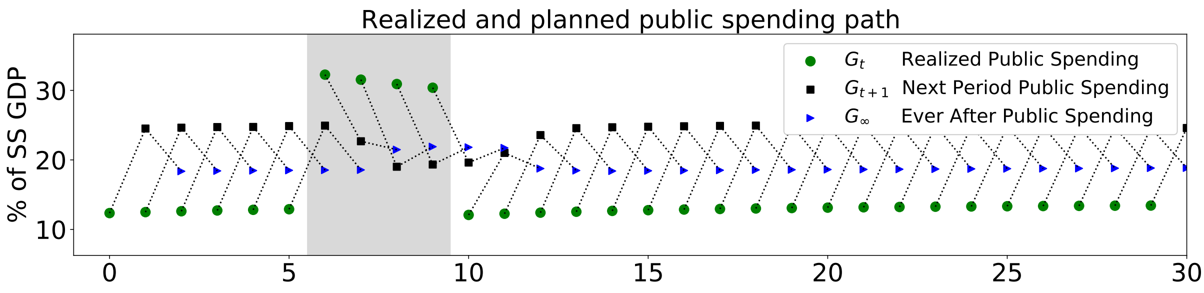

Figure 7: Realized and planned public spending

When the shock is low, the government sets low *current* public spending. *Ceteris paribus*, lower current spending entails fewer bond emissions and so households save through capital accumulation. Symmetrically, high public spending is the main driver of the capital stock decrease. The *next-period* spending also plays a role. *Ceteris paribus*, lower next-period spending entails fewer bond emissions in the next period. So households expects higher returns in the near future and save more to benefit from these higher returns on savings. Higher current saving implies more capital in the next period.

Shock persistence. In this paragraph, I discuss the effects of varying the persistence of shocks. I first consider the case of independent and identically distributed (iid) shocks, that is the absence of persistence. With iid shocks, the government knows that, when the shock is low, the next-period shock is more likely to be high compared to the case with persistent shocks. As a consequence, the motive for accumulating a capital buffer is stronger. When the shock is high, the government knows that the next-period shock is more likely to be low compared to the case with persistent shocks. As a consequence, the capital decreases more quickly. The opposite reasoning applies when the shocks are *more* persistent. On the whole, the less persistent the shocks are, the more the capital stock varies. This is displayed in Figure 8.

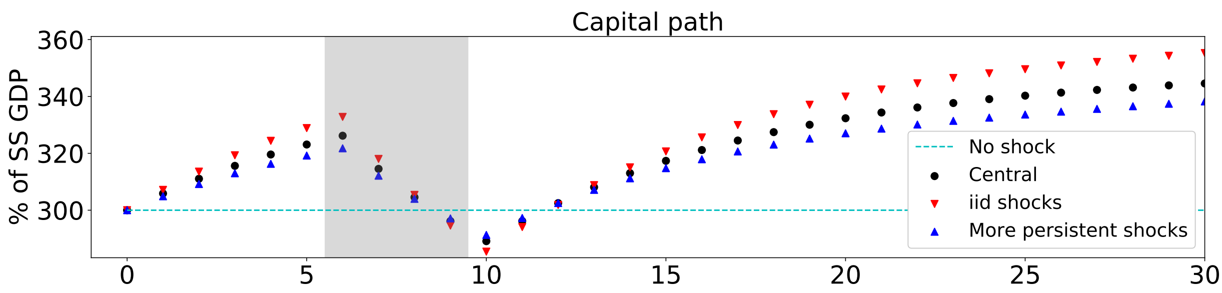

Figure 8: Capital and shock persistence

To drive the capital stock as the government wishes, the government sets the *next-period* spending to front-load or back-load future public spending. Compared to the case with persistent shocks (see Figure 7), Figure 9 shows that the front-loading and the back-loading of future public spending are more dramatic with iid shocks.

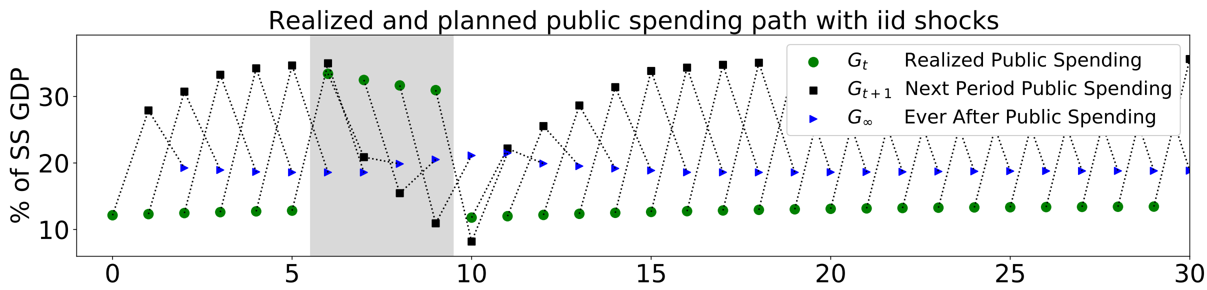

Figure 9: Realized and planned public spending with iid shocks

Tax distortions. Since distortion costs are convex in the labor tax rate, it is optimal to keep the labor tax as smooth as possible. Ideally, the labor tax should be constant across periods and across realizations of shocks.[15] Under the No Regret rule, the tax rate on labor is rather smooth (The maximum change in tax rate is $\approx 2.5\%$ when the shock goes from low to high or from high to low.) because the government uses debt to smooth tax revenues.

The capital tax distortion is very small. Figure 10 shows the implicit wedge due to the capital tax, that is the implicit solution of the following equation,

$$u'(C_t) \;=\; (1-x)\,\mathbb{E}_t[\,F_K(K_{t+1}, L_{t+1})u'(C_{t+1})\,]$$

This wedge is a measure of the distortion in capital investment due to the capital tax. This wedge is very small, always below 0.2% in absolute values. This implies that that capital investment levels are hardly distorted by that capital tax. The tax on bonds yields similar results.

[15]The labor tax under the optimal contingent policy is constant across periods and across realizations of shocks

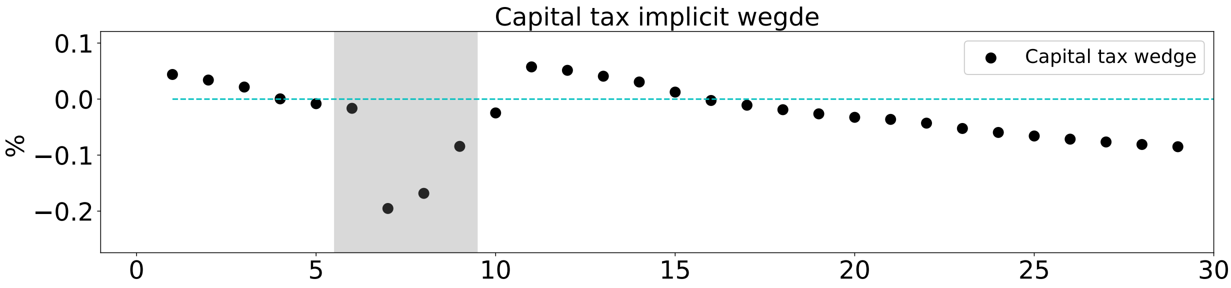

Figure 10: Capital tax implicit wegde

4.4 Welfare

Three mechanisms. Under the No Regret rule, the government uses three mechanisms to adapt to shocks: 1) it varies the labor tax, 2) it runs budget surpluses or deficits, and 3) it uses the capital stock as a buffer. Are these mechanisms equivalently useful to adapt to spending shocks? I froze one or two mechanisms and computed the associated welfare losses in consumption equivalent.

WELFARE DIFFERENCES (in consumption equivalent)	
Optimal No Regret policy	-
... With Constant Capital i.e. $K_t = K_{SS}$	-0.05%
... With Balanced Budget i.e. $B_t = 0$	-0.04%
... And Constant Capital i.e. $K_t = K_{SS}$	-0.49%
... With Constant labor Tax i.e. $\tau_t^L = \tau_{SS}^L$	-0.08%
... And Constant Capital i.e. $K_t = K_{SS}$	-0.71%
Optimal Constant Policy	-1.20%

The two main takeaways are the following. First, with this specification, the three mechanisms appear similarly useful to adapt to shocks. Second, freezing one out of the three mechanisms is not very costly in terms of welfare.

Optimal contingent policy. A key result is that the difference in welfare levels achieved by the optimal contingent policy and the optimal No Regret policy is very small, less than 0.01% in consumption equivalent.

5 Near-Rational Expectations

In this section, I study robustness to Near-Rational Expectations *à la* Woodford (2010). The government recognizes that households' expectations about shocks are not necessary equal to its own expectations. The government is *uncertain* about the households' beliefs and wants to implement a policy *robust* to this uncertainty. I establish that if the robustness requirement is large enough, the optimal allocation is implementable under the No Regret rule. In other words, even if the commitment to a contingent policy is available, the government is not worse off under the No Regret rule.

5.1 The robust problem

Households' beliefs about the probability distribution of shocks θ are described by a probability distribution denoted p^B, which is not necessary equal to the probability distribution used by the government denoted p. There is a set of probability distributions $\mathcal{P}$ such that the government wishes to choose a policy that will be as good as possible for the worse belief p^B in the set $\mathcal{P}$. The larger the set $\mathcal{P}$, the larger the robustness required by the government.

In this section, I assume that commitment to a contingent policy is possible. Since households' beliefs p^B are unknown, the government cannot predict households' decisions and the corresponding tax revenues. So instead of writing a contingent policy which is a function of time t and the realization of shocks θ^t, the government may write a contingent policy which is also function of past households' decisions without being redundant. The timing is the following. At t=0, the government announces its policy such that, government expenditures, labor tax, capital tax, and next period contingent returns on bonds at each time t are functions of the realization θ^t and possibly of all households' decisions taken before t (i.e. $C^{t-1}, L^{t-1}, K^t, B^t$). The price of contingent bonds P_t may vary depending on households' beliefs, so the quantity sold B_{t+1} is mechanically set such that the government can pay for its current expenditures G_t. Then, the households' beliefs p^B are chosen in the set $\mathcal{P}$ to *minimize* the expected utility computed with the government's expectations p. I assume that the set $\mathcal{P}$ is a closed convex set and that the probability distribution used by the government p is interior. I also impose that the set of beliefs $\mathcal{P}$ has a Markov structure as defined below:

Assumption 1. ***(Markov structure of the set of beliefs $\mathcal{P}$)***
For the N possible values of θ_t, there are N sets of conditional beliefs denoted $\mathcal{P}(\theta_t)$ such that:

$$\mathcal{P} = \{ \ p^B \ \mid \textit{for all } \theta^t, \ p^B(\cdot|\theta^t) \in \mathcal{P}(\theta_t) \ \}$$

where $p^B(\cdot|\theta^t)$ is the conditional probability distribution of θ_{t+1}.

This assumption implies that the choice of conditional beliefs $p^B(\cdot|\theta^t)$ is not a function of the choice of beliefs of the distribution of θ^t but only a function of θ_t This assumption limits the complexity of the dynamic strategic interaction between the benevolent player and the malevolent player who selects the beliefs p^B.

Information structure. I assume that households all have the same beliefs p^B and that p^B is common knowledge among them. Knowing the contingent policy and the beliefs p^B, households are able to predict all variables for any realization of θ.[16]

The max-min problem. The government is *benevolent* and *paternalistic* in the sense that it *maximizes* the expected utility computed *with its own expectations* p. The government's problem is the following:

$$\begin{aligned}\max_{Policy}\quad \min_{p^B\in\mathcal{P}}\quad & \mathbb{E}_0[\sum_{s=0}^{\infty}\beta^t\,[\,u(C_t(\theta^t,p^B)) - v(L_t(\theta^t,p^B)) + \theta_t w(G_t(\theta^t,p^B))\,]\,] \\ \text{s.t.}\quad & C_t(\theta^t,p^B) + G_t(\theta^t,p^B)) + K_{t+1}(\theta^t,p^B)) \leq F(K_t(\theta^{t-1},p^B)),L_t(\theta^t,p^B)) \\ \text{to}\quad & \{C_t(\theta^t,p^B),L_t(\theta^t,p^B),K_{t+1}(\theta^t,p^B),B_{t+1}(\theta^t,p^B)\}_t \text{ optimal decisions with belief } p^B\end{aligned}$$

This max-min problem is difficult to solve because one has to define the policy for all realizations of beliefs. Fortunately, it can be rewritten in a simpler form as this next theorem shows.

Theorem 1. ***(Robust primal sequential approach)***
The solution to the following problem solves the max-min problem

$$\begin{aligned}\mathring{V}(K_0,A_0,\theta_0) = \max_{\substack{\{C_t(\theta^t),L_t(\theta^t),\\ G_t(\theta^t),K_{t+1}(\theta^t)\}_{t\geq 0}}}\quad & \mathbb{E}_0[\sum_{t=0}^{\infty}\beta^t\,[\,u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t))\,]\,] \qquad (12)\\ \text{s.t.}\quad C_t(\theta^t) + G_t(\theta^t) + K_{s+1}(\theta^t) \quad & \leq \quad F(K_t(\theta^{t-1}),L_t(\theta^t)) \quad \textit{for all } \theta^t \\ \text{to}\quad A_0 \quad \leq \quad & \min_{p^B\,\in\,\mathcal{P}}\ \mathbb{E}_0[\sum_{t=0}^{\infty}\beta^t\,[\,u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t)\,]\,\frac{p^B(\theta^t)}{p(\theta^t)}\,]\end{aligned}$$

The proof is in the Appendix.

I denote $\{C_t^\star(\theta^t),L_t^\star(\theta^t),G_t^\star(\theta^t),K_{t+1}^\star(\theta^t)\}_{t=0}^{\infty}$ the contingent allocation that solves the maximization problem and $p^\star$ the beliefs that solve the minimization problem.

Tax policy. I now turn to the implementation of the allocation $\{C_t^\star(\theta^t),L_t^\star(\theta^t),G_t^\star(\theta^t),K_{t+1}^\star(\theta^t)\}_{t=0}^{\infty}$ with a contingent policy. I establish that the allocation is implementable for all beliefs $p^B\in\mathcal{P}$.

[16] An alternative assumption would be to assume that the government commits to a plan of contingent net prices independent of households' past decisions. Each atomistic household would not need to compute the others' decisions to take the optimal decisions for herself.

Lemma 5. ***(Robust optimal policy)***
For any possible beliefs $p_B \in \mathcal{P}$*, it is possible to implement the allocation* $\{C_t^\star(\theta^t), L_t^\star(\theta^t), G_t^\star(\theta^t), K_{t+1}^\star(\theta^t)\}_{t=0}^\infty$ *with the contingent policy* $\{G_t(\theta^t), \tau_t^L(\theta^t), \tau_t^K(\theta^t), R_t^B(\theta^t), T_t(\theta^t, B_t), \}_{t=0}^\infty$ *defined as follows:*

- $G_t(\theta^t) \equiv G_t^\star(\theta^t)$
- $\tau_t^L(\theta^t) \equiv 1 - \frac{v'(L_t^\star(\theta^t))}{F_L(K_t^\star(\theta^{t-1}), L_t^\star(\theta^t))u'(C_t^\star(\theta^t))}$
- $\tau_t^K(\theta^t) \equiv 1 - \frac{u'(C_{t-1}^\star(\theta^{t-1}))}{\beta F_K(K_t^\star(\theta^{t-1}), L_t^\star(\theta^t))u'(C_t^\star(\theta^t))}$
- $\tau_t^B(\theta^t) \equiv 1 - \frac{1}{\beta}\frac{u'(C_{t-1}^\star(\theta^{t-1}))}{u'(C_t^\star(\theta^t))} \frac{A_t^\star(\theta^t) - \frac{u'(C_{t-1}^\star(\theta^{t-1}))K_t(\theta^{t-1})}{\beta}}{\mathbb{E}_{t-1}[\ [A_t^\star(\theta^t) - \frac{u'(C_{t-1}^\star(\theta^{t-1}))K_t(\theta^{t-1})}{\beta}]\frac{p^\star(\theta^t|\theta^{t-1})}{p(\theta^t|\theta^{t-1})}\ ]}$
- $T_t(\theta^t, P_t) \equiv B_{t+1}^\star(\theta^t)[\ P_t - 1\ ] \quad \geq 0$

Where $A_t^\star(\theta^t) \equiv \mathbb{E}_t[\sum_{s=t}^\infty \beta^{s-t}\ [\ u'(C_s(\theta^s))C_s(\theta^s) - v'(L_s(\theta^s))L_s(\theta^s)\ ]\ \frac{p^\star(\theta^s|\theta^t)}{p(\theta^s|\theta^t)}\ ]$
And where $B_t^\star(\theta^{t-1}) \equiv \frac{\beta\mathbb{E}_{t-1}[\ A_t^\star(\theta^t)\frac{p^\star(\theta^t|\theta^{t-1})}{p(\theta^t|\theta^{t-1})}]}{u'(C_{t-1}^\star(\theta^{t-1}))} - K_t^\star((\theta^{t-1}))$

The proof is in the Appendix.

The intuition for this result is the following. The *worst* households' beliefs are the beliefs that minimize the bond prices, so any other beliefs yield higher prices and lower quantities of bonds sold. Since the government sells less bonds, positive lump-sum transfers are enough to make sure that households have the same wealth whatever their beliefs.[17]

Break-even beliefs. As mentioned above, the larger the set $\mathcal{P}$, the larger the robustness required by the government. This subsection establishes that there are beliefs denoted p^{BE} such that, if $p^{BE} \in \mathcal{P}$, then the optimal robust allocation can be implemented under the No Regret rule.

The optimal contingent allocation under the No Regret rule (i.e. solution to the problem (8), denoted $\{C_t^{NR}(\theta^t), L_t^{NR}(\theta^t), G_t^{NR}(\theta^t), K_{t+1}^{NR}(\theta^t)\}_{t=0}^\infty$ is associated with Largrange multipliers for the implementability constraints $\{\gamma_t^{NR}(\theta^t)\}_{t=0}^\infty$. First order conditions and envelope theorems gives that for any realization θ^t

$$\gamma_t^{NR}(\theta^t) = \frac{\theta_t w'(G_t^{NR}(\theta^t)) - v'(L_t^{NR}(\theta^t))}{v''(L_t^{NR}(\theta^t))L_t^{NR}(\theta^t) - v'(L_t^{NR}(\theta^t))}$$

[17]Of course, when taxes are positive, positive lump-sum transfers are not optimal so the government could achieve higher welfare with beliefs different than $p^\star$.

These multipliers $\{\gamma_t^{NR}(\theta^t)\}_{t=0}^{\infty}$ represent the shadow prices of the households' wealth $\{A_t^{NR}(\theta^{t-1})\}_{t=0}^{\infty}$. Under the No Regret rule, it is not possible to transfer households' wealth from realizations of shocks θ^t where its shadow value $\gamma_t^{NR}(\theta^t)$ is high to realizations of shocks where it is low. These transfers can be realized with contingent bonds. But the prices of these bonds may vary with beliefs p^B. The following beliefs, which I call break-even beliefs and denote p^{BE}, are built such that the prices of contingent bonds are low enough to bring zero marginal welfare gains. For any realization of shocks θ^t, the break-even beliefs p^{BE} have the following conditional probability distribution for the next period shock θ_{t+1}:

$$p_{t+1}^{BE}(\ \theta_{t+1} \mid \theta^t\ ) \equiv p_{t+1}(\ \theta_{t+1} \mid \theta_t\ ) \frac{\gamma_{t+1}^{NR}(\theta^{t+1})}{\mathbb{E}_t[\ \gamma_{t+1}^{NR}(\theta^{t+1})\ ]}$$

The break-even beliefs p^{BE} are deviations of the true distribution of shocks p. With the break-even beliefs p^{BE}, shocks which yield high shadow prices of the values of household's wealth are more likely. So using contingent bonds with low payoffs with these high shocks can only be sold at prices too low to be worth emitting. This gives us the main result of this section:

Theorem 2. ***(Near-Rational expectations and the No Regret rule)***
If the government robustness requirement is such that the break-even beliefs are in the interior of the set of possible beliefs (i.e. $p^{BE} \in \dot{\mathcal{P}}$), then the optimal robust allocation is implementable under the No Regret rule.

The proof is in the Appendix.

This result establishes that, when the government's concern for robustness to Near-Rational Expectations is high enough, the government never uses contingent bonds to transfer households' wealth from one realization of the next period shock to another. Numerical simulations show that the break-even beliefs p^{BE} are close to the true distribution p (see figure 11). Thus even a low concern for robustness to Near-Rational Expectations is enough to make contingent bonds useless.

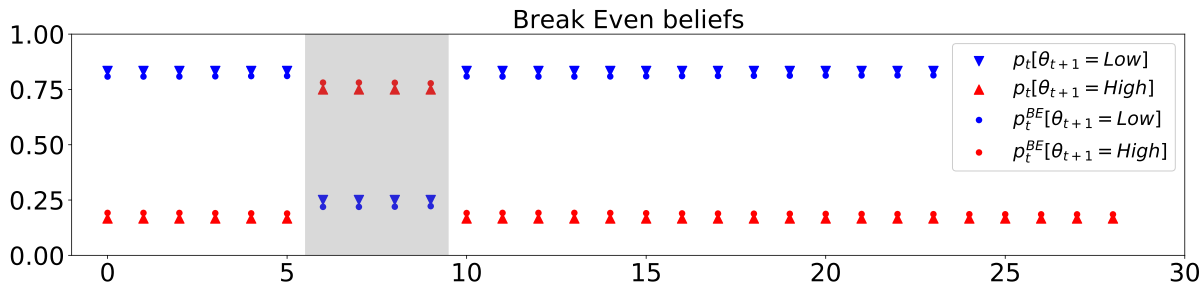

Figure 11: Break Even beliefs

6 Heterogeneity

In this section, I introduce heterogeneity among households. In the previous sections, I have studied how the No Regret rule tackled several classic time-inconsistency problems: capital taxation, sovereign default and bond prices manipulation. Without capital, nor government bonds in the economy, these time-inconsistency problems are absent. As a consequence, full discretion yields the optimal contingent policy. With heterogeneous households other time-inconsistency problems appear. First, with wealth heterogeneity, a government with full discretion caring for redistribution is tempted to tax wealthy households to redistribute towards poorer households. Anticipating this, rational households avoid saving and mutually beneficial exchanges of consumption across time through lending and borrowing will not take place. The government may also use the information on skills conveyed in past labor supply decisions to heavily tax more productive households. Anticipating this, all households pool and make identical labor decisions, which is detrimental to welfare. In this section, I introduce wealth and skill heterogeneity and show how the No Regret rule applies to this setup. Under the No Regret rule, there exist wealth heterogeneity and labor supply heterogeneity, which shows that both time inconsistency problems are well addressed.

6.1 Wealth heterogeneity

Let us first introduce wealth heterogeneity: households still have the same preferences but may have different private wealth denoted a. At each time t and for each household i, I define a_t^i as the wealth with which household i enters period t i.e. before returns and taxes are paid. Under the No Regret rule, the composition of wealth is irrelevant since all assets (capital, government bonds and households' debt) have identical net of tax returns. Wealth levels are observed by the government. Depending on her wealth $a_{t_R}^i$, any household i expects to get a total utility denoted $U(a_{t_R}^i)$ if the government's policy is never reformed. With a reform at time t_R, the new total expected utilities are denoted $\tilde{U}(a_{t_R}^i)$. The next result shows that, to be No Regret, it is sufficient and necessary for a reform to shift households' total expected utility distribution in parallel. In other words, all households, whatever their wealth levels, should see their expected utilities shifted by the same amount with the reform.

Theorem 3. ***(No Regret reform with wealth heterogeneity)***
The reform at time at t_R is a No Regret fiscal reform if and only if there exists a constant H such that for all asset level a, $U(a) = \tilde{U}(a) + H$

Without heterogeneity, the desire for flexible fiscal policies comes from changes in *preferences regarding government expenditures*. With heterogeneity, the desire for flexible fiscal policies may also come from changes in *preferences regarding redistribution*. This theorem, however, establishes that a No Regret reform is not able to achieve more (nor less) redistribution across households who differ only by their wealth levels.

Equal Sacrifice principle. According the Equal Sacrifice principle, households facing a common burden (here an increase in government expenditures) should share it such that the utility loss is equal among them. This principle has a clear normative appeal of equity. This paper provides another rationale: the Equal Sacrifice principle should be respected along the wealth heterogeneity dimension to make sure that households do not regret their past saving efforts. A symmetric Equal Benefit principle should be respected when there is a common benefit to be shared.

6.2 Skill heterogeneity

I now introduce skill heterogeneity i.e. the disutility of labor becomes skill dependent $v(L,\omega)$ where L is the *efficient* labor effort and $\omega \in [\underline{\omega}, \overline{\omega}]$ is the skill. The disutility of labor is decreasing with the skill i.e. $\frac{\partial v(L,\omega)}{\partial \omega} < 0$. I also assume that $\frac{\partial^2 v(L,\omega)}{\partial \omega \partial L} < 0$.[18] A household's *type* is her skill ω and her initial wealth level a_0. I denote $f(\cdot,\cdot)$ the joint density of types. The marginal density of skill is denoted $f_\omega(\cdot)$. To keep the model tractable with two heterogeneity dimensions, I take a mechanism design approach where the government directly selects consumption $C(\omega, a_0)$ and efficient labor effort $L(\omega, a_0)$ for each *type* (ω, a_0).

I make two simplifying assumptions. First, consumption levels and labor efforts are time-invariant in the absence of reform. Thus, the flow utility for each type is

$$U(\omega, a_0) \equiv u(C(\omega, a_0)) - v(L(\omega, a_0), \omega)$$

Second, there is no capital and production equals effective labor. The resource constraint is

$$\int C(\omega, a_0) f(\omega, a_0) d\omega da_0 \; + \; G \; \leq \int L(\omega, a_0) f(\omega, a_0) d\omega da_0 \tag{13}$$

No Regret fiscal reform. A No Regret fiscal reform is implemented at time t_R. The new allocation is denoted $\tilde{C}(\cdot,\cdot), \tilde{L}(\cdot,\cdot)$ and the new utility flows are denoted $\tilde{U}(\cdot,\cdot)$. The government does not observe skills ω. Truthful reporting of skills by households implies the following constraint on $\tilde{L}(\cdot,\cdot)$, $\tilde{U}(\cdot,\cdot)$.

$$\frac{\partial \tilde{U}}{\partial \omega}(\omega, a_0) = -\frac{\partial v(\tilde{L}(\omega, a_0), \omega)}{\partial \omega}$$

In the previous subsection, the theorem (3) established that, when a No Regret fiscal reform is implemented, all households with the same skill must have the same utility

[18]This inequality implies the Single Crossing property

loss or gain, whatever their wealth levels. This means that there exists a function $\tilde{H}(\cdot)$ such that, for all skill ω and for all initial wealth level a_0,

$$\tilde{U}(\omega, a_0) = U(\omega, a_0) + \tilde{H}(\omega)$$

Thus, when considering changes in the weighted average of utility levels, it is without loss of generality to use Pareto welfare weights that are only *skill dependent* rather than *wealth and skill dependent*. These stochastic weights are denoted $\Omega_t(\cdot)$. At time t_R, the government's problem is

$$\begin{aligned}
\max_{\substack{\tilde{C}(\cdot,\cdot),\tilde{L}(\cdot,\cdot),\\ \tilde{U}(\cdot,\cdot),\tilde{H}(\cdot),G}} \quad & \int \Omega_{t_R}(\omega)\tilde{H}(\omega)f_\omega(\omega)d\omega \ + \ \theta w(G) \\
\text{subject to} \quad & \int \tilde{C}(\omega, a_0)f(\omega, a_0)d\omega da_0 \ + \ G \ \leq \int \tilde{L}(\omega, a_0)f(\omega, a_0)d\omega da_0 \\
\text{to} \quad & \tilde{U}(\omega, a_0) = U(\omega, a_0) + \tilde{H}(\omega) \\
\text{to} \quad & \frac{\partial \tilde{U}}{\partial \omega}(\omega, a_0) = -\frac{\partial v(\tilde{L}(\omega, a_0), \omega)}{\partial \omega} \\
\text{and to} \quad & \tilde{U}(\omega, a_0) = u(\tilde{C}(\omega, a_0)) - v(\tilde{L}(\omega, a_0), \omega)
\end{aligned}$$

6.3 Tax implementation

In this subsection, I show how the allocation solution to the previous problem can be implemented using taxes on labor incomes and on private wealth.

General case. To implement the new allocation $\tilde{L}(\cdot,\cdot)$, $\tilde{U}(\cdot,\cdot)$ the government uses *skill dependent* wealth tax schedules at the time of the reform t_R and *wealth dependent* labor tax schedules at all times $t \geq t_R$. For each skill ω, the *skill dependent* wealth tax schedule is denoted $\tau^K_\omega(a_0)$. For each initial wealth a_0, the *wealth dependent* labor tax schedule is denoted $\tau^{\tilde{L}_{a_0}(\tilde{L})}$.

Since the allocation is time-invariant after and before the reforms, the households' Euler equation implies that the net return on wealth is always equal to β^{-1}. At the time of the reform t_R, the Ex Post Euler Equation is

$$u'(C(\omega, a_0)) = [1 - (\tau^K_\omega)'(a_0)]u'(\tilde{C}(\omega, a_0))$$

I assume that households without wealth are not taxed and so I can integrate the previous expression and get for any wealth a_0

$$\tau^K_\omega(a_0) = a_0 - \int_0^{a_0} \frac{u'(C(\omega, s)}{u'(\tilde{C}(\omega, s))}ds$$

The skills ω, however, are not directly observed by the government. Using $\omega_{a_0}(\cdot)$, the inverse function of the increasing functions $\omega \mapsto L(\omega, a_0)$, one can rewrite the *skill dependent* tax schedules on wealth $\tau_\omega^K(a_0)$ as *labor dependent* tax schedules on wealth $\tau_L^K(a_0)$ a function of the *labor effort* L

$$\tau_L^K : a_0 \mapsto \begin{cases} a_0 - \int_0^{a_0} \frac{u'(C(\omega_{a_0}(L),s)}{u'(\tilde{C}(\omega_{a_0}(L),s))} ds & if \;\; L \in L([\underline{\omega}, \overline{\omega}], a_0) \\ a_0 & otherwise \end{cases}$$

So, after the reform, for each type (ω, a_0), the new wealth level becomes $a_{t_R} \equiv a_0 - \tau_L^K(a_0)$.

Using the households' budget constraints, I get that, for each type (ω, a_0), the labor tax at each time $t \geq t_R$ is

$$\tau^{\tilde{L}_{a_0}(\omega) = \tilde{L}(\omega,a_0) - \tilde{C}(\omega,a_0) + (1/\beta - 1)\tau_\omega^K(a_0)}$$

Using $\tilde{\omega}_{a_0}(\cdot)$, the inverse function of the increasing function $\omega \mapsto \tilde{L}(\omega, a_0)$, one can rewrite the *wealth dependent* labor tax schedule for each after-tax wealth a_{t_R}.

$$\tau^{\tilde{L}_{a_{t_R}} : \tilde{L} \mapsto} \begin{cases} \tilde{L} - \tilde{C}(\tilde{\omega}_{a_0}(\tilde{L}), a_0) + (1/\beta - 1)a_{t_R} & if \;\; \tilde{L} \in \tilde{L}([\underline{\omega}, \overline{\omega}], a_0) \\ \tilde{L} & otherwise \end{cases}$$

Note that the *wealth dependent* labor tax schedule is a function of the net of tax wealth *at the time of the reform* a_{t_R}. Households cannot change their labor tax schedules by changing their wealth *after the reform*.

Unique labor tax schedule. In the previous paragraph, I described the taxes on labor and capital used to implement the desired allocation $\tilde{L}(\cdot,\cdot)$, $\tilde{U}(\cdot,\cdot)$. It is also possible to *first* choose all the *wealth dependent* labor tax schedules and to *then* find *skill dependent* wealth tax schedules to make sure the reform abides by the No Regret rule. As an interesting special case, it is possible to use a *wealth independent* labor tax schedule i.e. a *unique* labor tax schedule for all households.

6.4 More heterogeneity dimensions

It should be noted that there are numerous heterogeneity dimensions with economic relevance (e.g. education, discount preferences, altruism, age, the elasticity of intertemporal substitution, the Frish elasticity etc) and that a government with limited information and simple fiscal instruments cannot announce a reform which entails no regret for all households. However, the No Regret fiscal rule can be adapted to more realistic environments.

According to the lemma (1), the best strategy for households is to take decisions as if the current policy is never reformed and to (rightfully) trust the government to never make them regret this strategy. So, if at time t_A a government announces a No Regret reform in advance, that is before the reform starts to be implemented at time t_R, then households do not modify their decisions between t_A and just before t_R. In particular, there is no consumption jump at time t_A. This is characterization is more convenient than the absence of regret requirement.

In more realistic environments, one can divide society into homogeneous groups of households and impose that a government under the No Regret rule could implement any reform provided there is no jump in average consumption within each group when a reform is announced.

7 Conclusion

The main contribution of this paper is to introduce No Regret fiscal reforms and the No Regret rule. The No Regret rule is simple. First, there is no need to *specify in advance* the set of policies available to the government as some rules do (Halac & Yared, 2014; Sleet, 2004). Second, the No Regret rule itself is non-contingent, the rule does not vary with shocks. I have shown that the small welfare loss compared to the optimal contingent policy is due to the fact that the government cannot get full insurance through contingent bonds. Both approaches yield very similar policies and allocations. A direct implication is that, although assuming commitment to a contingent policy is technically unrealistic, this assumption is not *critical*. Indeed, replacing this assumption with the more realistic No Regret rule yields similar policies. Another benefit of the No Regret rule is that households do not have to take into account potential future reforms because none will make them regret their past decisions. I also make a methodological contribution, I study the government's problem that is a complex recursive problem since every reformed policy may be reformed again in the future. I use a primal approach to characterize the solution.

I extend the model to study optimal contingent policies robust to Near-Rational Expectations. I establish that the welfare difference between the optimal robust contingent policy and the optimal No Regret policy disappears when the robustness requirement is high enough. Finally, I introduce wealth and skill heterogeneity and show that, under the No Regret rule, redistribution is possible across skills but not across wealth levels.

Several avenues for future research are worth mentioning. First, in this paper, I assume that the government is infinitely sophisticated in the sense that it knows all future shocks and their joint probability distributions. It could be interesting to relax this assumption by introducing unforeseen contingencies. Contingent policies won't be able to adapt to these shocks whereas a government under the No Regret rule could react. This would make the No Regret rule an even more convincing alternative to full commitment to a contingent policy. Note that the No Regret rule also allows for the introduction of new policy tools that were not foreseen (e.g. a carbon tax).

Second, in this paper, changes in the preference for public spending are a one-catch-all way to model exogenous shocks. But changes in the government's preferences may also result from the political process (e.g. elections) and so successive governments may have very different preferences. This could exacerbate the temptation to deviate from the *initially planned* contingent policy. Indeed, why would a government respect a contingent policy that goes strongly against their preferences? The No Regret rule, however, provides discretion and successive governments may adapt their policies to their preferences. The need to deviate from the No Regret rule is less stringent, which makes the No Regret rule stronger than full commitment to a contingent policy.

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8 Proofs

Necessity of (3), (4) and (5)

Assume $\{C^\infty, L^\infty, q^\infty, k^\infty\}$ is optimal, it is budget feasible so (2) hold.
We have assumed that the debt constraint was never binding so one can build small deviations to establish (3),(4),(5) by contradiction.

(5) : If $v'(L_t) < W_t u'(C_t)$ then there is $\varepsilon > 0$ such that a deviation with $\tilde{L}_t = L_t + \varepsilon$ and $\tilde{C}_t = C_t + W_t\varepsilon$ yields higher welfare to the households and (2) still holds, contradicting optimality. The reverse inequality can be ruled out with $\varepsilon < 0$.

(3) : If $u'(C_t) < \mathbb{E}_t[\ \beta R_{t+1} u'(C_{t+1})\ ]$ then there is $\varepsilon > 0$ such that a deviation with $\tilde{k}_{t+1} = k_{t+1} + \varepsilon$, $\tilde{C}_t = C_t - \varepsilon$ and $\tilde{C}_{t+1} = C_{t+1} + R_{t+1}\varepsilon$ yields higher welfare to the households and (2) still holds, contradicting optimality. The reverse inequality can be ruled out with $\varepsilon < 0$.

(4) : $P_t < \frac{\mathbb{E}_t[\ \beta B_{t+1} u'(C_{t+1})\ ]}{u'(C_t)}$ then there is $\varepsilon > 0$ such that a deviation with $\tilde{q}_t = q_t + \varepsilon$, $\tilde{C}_t = C_t - \varepsilon P_t$ and $\tilde{C}_{t+1} = C_{t+1} + B_{t+1}\varepsilon$ yields higher welfare to the households and (2) still holds, contradicting optimality. The reverse inequality can be ruled out with $\varepsilon < 0$.

■

Lemma 3: Convergence to First Best

The first order condition with respect to A_{t+1} and the envelope condition with respect to A_t gives for all t

$$\gamma_t = \mathbb{E}_t[\gamma_{t+1}] \geq 0$$

Hence γ is a positive super-martingale and therefore must converge almost surely towards a random variable denoted $\overline{\gamma}$ (Doob's first martingale convergence theorem). Furthermore the higher the θ_t, the higher future θ_{t+s} ,[19] and so the higher the need to tax and the higher the multiplier of the implementability constraint γ_t. So, for a given $\gamma_{t-1} > 0$, the value of γ_t is strictly increasing with the realization of θ_t. This means that γ_t cannot be constant across different shocks θ_t unless γ_t is zero. But since $\{\gamma_t\}_t$ converges almost surely, it must converge towards zero.

■

Lemma ??: No Regret allocations insensitive to households beliefs

Thanks to lemma 2, the Ex Post Euler Equation holds at any time t, i.e. $u'(C_t) = \beta R_{t+1} u'(C_{t+1})$. This means that for any beliefs p^B,

$$u'(C_t) = \mathbb{E}_t^B[\ \beta R_{t+1} u'(C_{t+1})\ ] := \int_{\omega\in\Omega} \beta R_{t+1}(\omega) u'(C_{t+1}(\omega)) dp^B(\omega)$$

This means that households' behaviours is insensitive to their beliefs $\{p_i^B\}_i$. So, the equilibrium allocation associated to the No Regret policy plan is insensitive to households' beliefs.

[19] If $\theta_t \geq \tilde{\theta}_t$, then the random variable $\{\theta_{t+s}\}_{s>0}$ conditional on θ_t has first-order stochastic dominance over $\{\tilde{\theta}_{t+s}\}_{s>0}$ conditional on $\tilde{\theta}_t$. This is because the transition probability of θ is non decreasing

■

Theorem 1: (Robust primal sequential approach)

The value of the max-min problem is lower that the value of the min-max problem

$$\begin{aligned} \min_{p^B \in \mathcal{P}} \quad \max_{Policy} \quad & \mathbb{E}_0[\sum_{s=0}^{\infty} \beta^t \; [\; u(C_t(\theta^t, p^B)) - v(L_t(\theta^t, p^B)) + \theta_t w(G_t(\theta^t, p^B)) \;] \;] \\ \text{s.t.} \quad & C_t(\theta^t, p^B) + G_t(\theta^t, p^B)) + K_{t+1}(\theta^t, p^B)) \leq F(K_t(\theta^{t-1}, p^B)), L_t(\theta^t, p^B)) \\ \text{to} \quad & \{C_t(\theta^t, p^B), L_t(\theta^t, p^B), K_{t+1}(\theta^t, p^B), B_{t+1}(\theta^t, p^B)\} \text{ optimal decisions with belief } p^B \end{aligned}$$

But then the policy is defined after the beliefs. So it is enough for the government to define the allocation that will be realized with these beliefs and so the problem becomes

$$\min_{p^B \in \mathcal{P}} \quad \max_{\substack{\{C_t(\theta^t), L_t(\theta^t), \\ G_t(\theta^t), K_{t+1}(\theta^t)\}_{t \geq 0}}} \quad \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \; [\; u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t)) \;] \;]$$

$$\text{subject to} \quad C_t(\theta^t) + G_t(\theta^t) + K_{s+1}(\theta^t) \leq F(K_t(\theta^{t-1}), L_t(\theta^t)) \quad \text{for all } \theta^t$$

$$\text{and to} \quad A_0 \leq \; \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \; [\; u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t) \;] \; \frac{p^B(\theta^t)}{p(\theta^t)} \;]$$

Then, with the envelope theorem applied to the inner maximisation problem and using the fact that the Lagrangian multiplier associated to the implementability constraint is positive ($\lambda > 0$)

$$V(p^B) \quad \equiv \quad \max_{\substack{\{C_t(\theta^t), L_t(\theta^t), \\ G_t(\theta^t), K_{t+1}(\theta^t)\}_{t \geq 0}}} \quad \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \; [\; u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t)) \;] \;]$$

$$\text{subject to} \quad C_t(\theta^t) + G_t(\theta^t) + K_{s+1}(\theta^t) \leq F(K_t(\theta^{t-1}), L_t(\theta^t)) \quad \text{for all } \theta^t$$

$$\text{and to} \quad A_0 \leq \; \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \; [\; u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t) \;] \; \frac{p^B(\theta^t)}{p(\theta^t)} \;] \qquad (\lambda)$$

we get that the minimizing beliefs should also minimize

$$\mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \; [\; u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t) \;] \; \frac{p^B(\theta^t)}{p(\theta^t)} \;]$$

The min-max problem can then be rewritten

$$\min_{p^B \in \mathcal{P}} \quad \max_{\substack{\{C_t(\theta^t), L_t(\theta^t), \\ G_t(\theta^t), K_{t+1}(\theta^t)\}_{t \geq 0}}} \quad \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \; [\; u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t)) \;] \;]$$

$$\text{subject to} \quad C_t(\theta^t) + G_t(\theta^t) + K_{s+1}(\theta^t) \leq F(K_t(\theta^{t-1}), L_t(\theta^t)) \ \text{ for all } \theta^t$$

$$\text{and to} \quad A_0 \leq \min_{\tilde{p}^B \ \in \ \mathcal{P}} \ \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \ [\ u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t)\ ] \ \frac{p^B(\theta^t)}{p(\theta^t)} \ ]$$

The outer minimization is useless so the min-max problem is equal to

$$\max_{\substack{\{C_t(\theta^t), L_t(\theta^t), \\ G_t(\theta^t), K_{t+1}(\theta^t)\}_{t\geq 0}}} \quad \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \ [\ u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t))\ ]\ ]$$

$$\text{subject to} \quad C_t(\theta^t) + G_t(\theta^t) + K_{s+1}(\theta^t) \leq F(K_t(\theta^{t-1}), L_t(\theta^t)) \ \text{ for all } \theta^t$$

$$\text{and to} \quad A_0 \leq \min_{p^B \ \in \ \mathcal{P}} \ \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \ [\ u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t)\ ] \ \frac{p^B(\theta^t)}{p(\theta^t)} \ ]$$

The last step is to show that the solution to this last problem can be implemented with a contingent policy and this is the lemma 5

■

Lemma 5: (Robust optimal policy)

Assume that households have beliefs $p^B \in \mathcal{P}$. Let us show that it is optimal for households to follow $\{C_t^\star(\theta^t), L_t^\star(\theta^t), B_{t+1}^\star(\theta^t), K_{t+1}^\star(\theta^t)\}_{t=0}^{\infty}$.

First, the equation (5) holds for any θ^t.
Second, the equation (3) holds for any beliefs p^B The price of bonds $P_t(\theta^t)$ are given by the net of tax given and the equation (4) computed with the beliefs p^B:

$$P_{t-1}(\theta^{t-1}, p^B) = \frac{\mathbb{E}_{t-1}[\ \beta u'(C_t^\star(\theta^t))(1 - \tau_t^B(\theta^t))\frac{p^B(\theta^t|\theta^{t-1})}{p(\theta^t|\theta^{t-1})}]}{u'(C_{t-1}^\star(\theta^{t-1}))}$$

$$P_{t-1}(\theta^{t-1}, p^B) = \frac{\mathbb{E}_{t-1}[\ \beta u'(C_t^\star(\theta^t))\frac{1}{\beta}\frac{u'(C_{t-1}^\star(\theta^{t-1}))}{u'(C_t^\star(\theta^t))} \frac{A_t^\star(\theta^t) - \frac{u'(C_{t-1}^\star(\theta^{t-1}))K_t(\theta^{t-1})}{\beta}}{\mathbb{E}_{t-1}[\ [A_t^\star(\theta^t) - \frac{u'(C_{t-1}^\star(\theta^{t-1}))K_t(\theta^{t-1})}{\beta}]\frac{p^\star(\theta^t|\theta^{t-1})}{p(\theta^t|\theta^{t-1})}\ ]}\frac{p^B(\theta^t|\theta^{t-1})}{p(\theta^t|\theta^{t-1})}]}{u'(C_{t-1}^\star(\theta^{t-1}))}$$

$$P_{t-1}(\theta^{t-1}, p^B) = \frac{\mathbb{E}_{t-1}[\ [A_t^\star(\theta^t) - \frac{u'(C_{t-1}^\star(\theta^{t-1}))K_t(\theta^{t-1})}{\beta}]\frac{p^B(\theta^t|\theta^{t-1})}{p(\theta^t|\theta^{t-1})}\ ]}{\mathbb{E}_{t-1}[\ [A_t^\star(\theta^t) - \frac{u'(C_{t-1}^\star(\theta^{t-1}))K_t(\theta^{t-1})}{\beta}]\frac{p^\star(\theta^t|\theta^{t-1})}{p(\theta^t|\theta^{t-1})}\ ]}$$

So

$$P_{t-1}(\theta^{t-1}, p^\star) = 1$$

Using the definition of $B_t^\star(\theta^{t-1})$

$$P_{t-1}(\theta^{t-1}, p^B)B_t^\star(\theta^{t-1}) = \frac{\mathbb{E}_{t-1}[\ A_t^\star(\theta^t)\frac{p^B(\theta^t|\theta^{t-1})}{p(\theta^t|\theta^{t-1})}\ ] - \frac{u'(C_{t-1}^\star(\theta^{t-1}))K_t(\theta^{t-1})}{\beta}}{u'(C_{t-1}^\star(\theta^{t-1}))/\beta}$$

By the definition of $p^\star$ and thanks to the assumption (1), we have

$$\mathbb{E}_{t-1}[\ A_t^\star(\theta^t)\frac{p^B(\theta^t|\theta^{t-1})}{p(\theta^t|\theta^{t-1})}\ ] \geq \mathbb{E}_{t-1}[\ A_t^\star(\theta^t)\frac{p^\star(\theta^t|\theta^{t-1})}{p(\theta^t|\theta^{t-1})}\ ]$$

So

$$P_{t-1}(\theta^{t-1}, p^B)B_t^\star(\theta^{t-1}) \geq P_{t-1}(\theta^{t-1}, p^\star)B_t^\star(\theta^{t-1}) = B_t^\star(\theta^{t-1})$$

That is whatever the beliefs p^B, if $B_t^\star(\theta^{t-1}) \geq 0$ then the households are paying more for the bonds than when they have beliefs $p^\star$. Symmetrically, when they are borrowing money from the government ($B_t^\star(\theta^{t-1}) \leq 0$), they are getting more money for the same repayment.

This inequality also yields that transfers T_t are indeed positive. These transfers are such that households get back the money they loose compared to the case they have beliefs p^B. Thus, their budget constraints are exactly binding if they follow the allocation $\{C_t^\star(\theta^t), L_t^\star(\theta^t), B_{t+1}^\star(\theta^t), K_{t+1}^\star(\theta^t)\}_{t=0}^\infty$.

■

Theorem 2: (Near-Rational expectations and No Regret rule)

Thanks to the assumption (1), the set of possible beliefs $\mathcal{P}$ has a recursive structure. So, the problem (12) can be rewritten in a recursive form:

$$\begin{aligned} \mathring{V}(K_t, A_t, \theta_t) = \max_{\substack{C_t, L_t, G_t \\ K_{t+1}, A_{t+1}(\cdot)}} \quad & u(C_t) - v(L_t) + \theta_t w(G_t) \ + \ \beta \mathbb{E}_t[\ \mathring{V}(K_{t+1}, A_{t+1}(\theta_{t+1}), \theta_{t+1})\ ] \qquad (14) \\ \text{s.t.} \quad & C_t + G_t + K_{t+1} \leq F(K_t, L_t) \\ \text{to} \quad & A_t \ \leq \ u'(C_t)C_t - v'(L_t)L_t + \min_{p^B(\cdot\ |\ \theta_t)\ \in\ \mathcal{P}(\theta_t)} \beta \mathbb{E}_t[\ A_{t+1}(\theta_{t+1})\ \frac{p^B(\theta_{t+1}\ |\ \theta_t)}{p(\theta_{t+1}\ |\ \theta_t)}\ ] \end{aligned}$$

Let us show that the policy functions associated to the problem (8) and denoted $\{C^{NR}(\cdot,\cdot,\cdot), L^{NR}(\cdot,\cdot,\cdot), G^{NR}(\cdot,\cdot,\cdot), K_{+1}^{NR}(\cdot,\cdot,\cdot), A_{+1}^{NR}(\cdot,\cdot,\cdot)\}_{t=0}^\infty$ that are optimal under the No Regret rule are also solutions to the problem (12).

Let us consider a small deviation from these policy functions. Since these policy functions are optimal under the constraint that $A_{+1}(\cdot,\cdot,\cdot,\cdot)$ is *independent* of the next period shock, first order welfare gains can only be achieved by having $A_{+1}(\cdot,\cdot,\cdot,\cdot)$ *depend* on the next period shock. Accordingly, let us consider the deviation where for all $K_t, A_t, \theta_t, \theta_{t+1}$, and for a small non-negative ε,

$$A_{+1}(K_t, A_t, \theta_t, \theta_{t+1}) \ \equiv A_{+1}^{NR}(K_t, A_t, \theta_t) + \varepsilon\delta(K_t, A_t, \theta_t, \theta_{t+1})$$

where $0 \ < \ \|\delta(\cdot,\cdot,\cdot,\cdot)\|_{Max} \ \leq \ 1$.
Furthermore,

$$\frac{\partial \mathring{V}(K_{t+1}, A_{t+1}, \theta_{t+1})}{\partial A_{t+1}} = \frac{\partial V(K_{t+1}, A_{t+1}, \theta_{t+1})}{\partial A_{t+1}} + O(\varepsilon) = -\gamma^{NR}(K_{t+1}, A_{t+1}, \theta_{t+1}) + O(\varepsilon)$$

Since this deviation only affects the next period state variables and since by assumption the welfare gain achieved by the deviation is non-negative, we have

$$\mathbb{E}[\ (\ -\gamma^{NR}(K_{t+1}, A_{t+1}, \theta_{t+1}) + O(\varepsilon)\ )\ \varepsilon\ \delta(K_t, A_t, \theta_t, \theta_{t+1}) \mid \theta_t\ ] \geq 0$$

which implies

$$\mathbb{E}[\ \gamma^{NR}(K_{t+1}, A_{t+1}, \theta_{t+1})\ \delta(K_t, A_t, \theta_t, \theta_{t+1}) \mid \theta_t\ ] \leq 0$$

The deviation must also meet the constraints so

$$\min_{p^B(\cdot\ \mid\ \theta_t)\ \in\ \mathcal{P}(\theta_t)} \ \beta\mathbb{E}_t[\ \varepsilon\delta(K_t, A_t, \theta_t, \theta_{t+1})\ \frac{p^B(\theta_{t+1} \mid \theta_t)}{p(\theta_{t+1} \mid \theta_t)}\ ] \geq 0$$

The break-even beliefs p^{BE} are such that

$$p^{BE}(\theta_{t+1} \mid \theta_t) = p(\theta_{t+1} \mid \theta_t)\frac{\gamma^{NR}(K_{t+1}, A_{t+1}, \theta_{t+1})}{\mathbb{E}[\ \gamma^{NR}(K_{t+1}, A_{t+1}, \theta_{t+1} \mid \theta_t\ ]}$$

By assumption the break-even beliefs p^{BE} are in the set of possible beliefs so

$$\mathbb{E}[\ \gamma^{NR}(K_{t+1}, A_{t+1}, \theta_{t+1})\ \delta(K_t, A_t, \theta_t, \theta_{t+1}) \mid \theta_t\ ] \geq 0$$

And so,

$$\mathbb{E}_t[\ \varepsilon\delta(K_t, A_t, \theta_t, \theta_{t+1})\ \frac{p^{BE}(\theta_{t+1} \mid \theta_t)}{p(\theta_{t+1} \mid \theta_t)}\ ] = 0$$

Furthermore, the break-even beliefs are *interior*, so one can build a deviation of the break-even beliefs $\tilde{p}^{BE}$ such that

$$\mathbb{E}_t[\ \delta(K_t, A_t, \theta_t, \theta_{t+1})\ \frac{\tilde{p}^{BE}(\theta_{t+1} \mid \theta_t)}{p(\theta_{t+1} \mid \theta_t)}\ ] < 0$$

which implies that the deviation does not meet the constraints.

■