Ethical Voting in Heterogeneous Groups

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Abstract

Voting in large elections appears to be both ethically motivated and influenced by strategic considerations. A common way to capture this interplay postulates a rule-utilitarian calculus within groups of supporters. I compare this approach with a model of Kantian behavior, in which voters maximize their utility under a moral constraint of universalization. In line with the empirical evidence, the model predicts a higher turnout rate among supporters with more intense preferences, linking the ethical motive to the spatial theory of voting. The result contrasts with the rule-utilitarian logic in heterogeneous groups, according to which differences in the intensity of preferences should be irrelevant for participation.

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1 Introduction

Explaining voting in large elections has proven difficult. The instrumental model, with citizens moved only by the desire to affect the outcome, clashes against the negligible probability of being pivotal. Hence, even small voting costs bind the predicted turnout close to zero. To resolve the impasse, previous research has stressed the importance of voting out of civic duty (Blais 2000, François and Gergaud 2019, Blais and Daoust 2020). However, participation does vary with strategic considerations. For example, turnout is typically increasing in the expected closeness of the election, in line with a higher likelihood of affecting the outcome (Shachar and Nalebuff 1999, Fauvelle-Aymar and François 2006, Arnold 2018, Bursztyn et al. 2017). The challenge posed by the empirical evidence is thus to model the interaction between voters' ethical and strategic reasoning.

In two seminal contributions, Feddersen and Sandroni (2006) and Coate and Conlin (2004) proposed to do so through a *rule-utilitarian* model, which has arisen as a leading economic theory of voter turnout. Rule-utilitarian supporters set a turnout rule which maximizes aggregate utility. The rule takes the form of a threshold in the cost of voting, below which supporters should vote. Voting is thus both motivated by the utilitarian ethics and responsive to the characteristics of the election, since the threshold cost is endogenous. Postulating voters' reasoning as a rule-utilitarian calculation, however, is only one way in which the concern for fulfilling a civic duty can be understood.

In this paper, I examine voter turnout through an alternative framework, which builds on a requirement of universality for an ethical rule. In this framework, agents choose their behavior under the self-imposed constraint that the others behave in the same way as themselves. This reasoning is consistent with citizens expressing a concern about what would happen if everyone abstained. Economists typically call such behavior *Kantian*, because the process of universalization is reminiscent of Kant's categorical imperative (Laffont 1975). The key aspect is that Kantian behavior is in contrast to the standard notion of best-reply. From an agent's point of view, the strategies of the other players are not kept fixed but change as a function of the agent's own strategy. Unlike the rule-utilitarian approach, however, this reasoning does not involve any aggregation of welfare: under the constraint of universalization, agents maximize their individual utility rather than a social welfare function.

I explore the implications of Kantian behavior in a spatial setting, in which citizens have idiosyncratic preferences and form groups of supporters depending on the distance between their preferred policy and those proposed by the candidates. As a consequence, the groups are heterogeneous, because the intensity of preference differs among supporters of the same candidate. To deal with Kantian behavior in the presence of heterogeneity, I follow the theory of *Kantian Optimization* introduced by Roemer (2010, 2015, 2019). Agents evaluate any deviation by the consequences resulting if a deviation by the same multiplicative factor is taken collectively in their group. Hence, the model captures the ethical concern of voters who, when considering a reduction in their likelihood of voting, envision a reduction by the same proportion by the other supporters. The equilibrium is determined by the condition that no deviation by the same factor is profitable for any supporter in each group, given the voting behavior in the opposite group.

This type of Kantian reasoning yields substantial participation, comparable in size with that from the rule-utilitarian model. Moreover, the model predicts the probability of voting to be increasing in the intensity of preference, as measured by the utility differential from candidates' policies. The result is consistent with the fact that the ideological distance between voters and candidates affects not only the vote choice but also the likelihood of participating. For example, the previous literature has stressed that citizens are less likely to vote if they feel equally close to the competing candidates or too far from all of them (Enelow and Hinich 1984, Zipp 1985, Plane and Gershtenson 2004, Adams, Dow and Merrill 2006). By contrast, I show that the rule-utilitarian logic applied to heterogeneous agents does not suffice to generate this intuitive pattern.

The relation between the two ethical frameworks depends specifically on the assumption about the voting costs. If voting costs are fixed and identical for all citizens, the utilitarian calculus pins down the number of votes in each group but is silent on which members should provide these votes, since it is irrelevant for aggregate utility. Yet, the number of votes coincides with the one from the Kantian model, which specifies the individual probabilities of voting as a function of supporters' preferences. Hence, the Kantian model can offer a selection device for a rule on which rule-utilitarian supporters can coordinate. This complementarity breaks down if voting costs differ and are ex-ante unknown. In this case, the utilitarian logic imposes the same likelihood of voting (ex-ante) for supporters with different preferences. Indeed, aggregate voting costs are minimized by disregarding the heterogeneity in preferences. This result is at odds with the evidence of participation depending on the distance between voters and candidates, which is instead consistent with the Kantian model.

To illustrate further, note that the rule-utilitarian model builds on Harsanyi's (1980) thought experiment of choosing under a *veil of ignorance* about the cost of voting. In Feddersen and Sandroni (2006) and Coate and Conlin (2004), the cost of voting is the only dimension of (ex-post) heterogeneity, since all supporters of a candidate obtain the same benefit from the outcome of the election. Supporters are thus ex-ante identical and benefit equally from maximizing aggregate utility. In a spatial setting, however, voters' policy preferences represent the conflict of interests inherent to politics, and thus a heterogeneity for which the veil of ignorance is less plausible. In this case, the utilitarian maximization predicts aggregate turnout well but neglects the individual heterogeneity.[1] The Kantian model, instead, matches the properties of turnout at the individual level because voters maximize their individual utility. Therefore, idiosyncratic preferences, which determine the intensity of support for a candidate, are taken into account under the universalization constraint and yield heterogeneous turnout rates.

The analysis of cooperation through Kantian behavior has received attention by several scholars. Early references include Laffont (1975), Sugden (1984), and Bilodeau and Gravel (2004). More recently, Alger and Weibull (2013, 2016) and Alger, Weibull, and Lehmann (2020) have studied the evolutionary foundations of morality and have found that partially Kantian preferences - which they label *Homo Moralis* - stand out as evolutionarily stable. Alger and Laslier (2021) argue that Homo Moralis agents turn out at substantial rates even with small degrees of morality if the marginal cost of voting is low enough. In the framework of Homo Moralis, however, agents are always ex-ante identical. Instead, Roemer's Kantian optimization (2010, 2015, 2019) allows for differences among agents by considering deviations of the same type from any strategy profile. Roemer develops the theory in broader terms, as a cooperative protocol that yields Pareto-efficient outcomes. A small application to voter turnout is in Roemer (2010), in which however the existence of an equilibrium with non-zero participation is hindered by the discontinuity of voters' payoff functions in a deterministic model.[2]

[1] The focus is indeed on aggregate turnout in Bierbrauer, Tsyvinsky, and Werquin (2021), who study spatially the joint determination of turnout and candidates' platforms, as well as in the models by Jorgenson and Saavedra (2018), Ali and Lin (2013), and Levine and Mattozzi (2020).

[2] See also Roemer (2006) for an application to the financing of political campaigns by party members, and De Donder and Roemer (2016) for an application to lobbying. Grafton, Kompas and Long (2017) and Eichner and Pethig (2020) present models with a mix of Kantian and Nashian agents, on climate change mitigation and tax competition respectively. On the empirical side, De Donder et al. (2021) show that vaccination patterns in Europe and North America are more consistent with Kantian behavior rather than Nashian. Kantian optimization is also discussed by Maniquet (2019), Laslier (2020) and Sher (2020).

The remainder of the paper is organized as follows. Section 2 presents the setting and voters' ethical reasoning. Section 3 shows the main results and the comparison with the rule-utilitarian approach. Section 4 discusses the endogenization of candidates' policies, the possibility of amending the rule-utilitarian calculus, as well as the Kantian label that economists assign to the universalization principle. Section 5 concludes.

2 Voting in a spatial setting

The policy space is the interval $[0, 1]$. Two candidates, A and B, compete in an election by proposing policies a and b. A continuum of citizens is distributed according to a density function $f(z)$, where z denotes their preferred policy (or bliss point). Each citizen z evaluates any policy x with a utility function $u(z-x)$, denoted shortly $u_z(x)$. That is, the policy benefits $u_z(a)$ and $u_z(b)$ depend on the distance between the preferred policy z and the proposed policies a and b.

Assumption 1. *The utility function $u_z(x)$ is continuous and single-peaked around the preferred policy z. The function u is the same for all citizens z.*

Two groups of supporters for the two candidates are then formed endogenously as $\mathcal{Z}_A = \{z : u_z(a) > u_z(b)\}$ and $\mathcal{Z}_B = \{z : u_z(b) > u_z(a)\}$. Assumption 1 guarantees that the two groups are separated on the policy space by an indifferent citizen z^*. If without loss of generality $a \leq b$, then $\mathcal{Z}_A = [0, z^*)$ and $\mathcal{Z}_B = (z^*, 1]$.[3]

Citizens' vote choice is sincere in favor of the preferred candidate, because voting only affects whether policy a or b is implemented. The participation behavior is then described by two functions $p_A(z) : \mathcal{Z}_A \to [0, 1]$ and $p_B(z) : \mathcal{Z}_B \to [0, 1]$, which give the probability of voting for each supporter in each group and are derived through the ethical calculus described in the next subsection. The number of votes is obtained from aggregating these functions within each group, i.e. $v_A = \int_{\mathcal{Z}_A} p_A(z)f(z)dz$ and $v_B = \int_{\mathcal{Z}_B} p_B(z)f(z)dz$. As a function of the number of votes v_A and v_B, the consequences of the election are probabilistic: policy a is implemented with some probability $P(v_A, v_B)$, while policy b is implemented with probability $1 - P(v_A, v_B)$.

Assumption 2. *(i) $P(v_A, v_B)$ is increasing in v_A, decreasing in v_B, and equal to $\frac{1}{2}$ if $v_A = v_B$. (ii) $P(v_A, v_B)$ is continuous, concave in v_A, and convex in v_B.*

[3]If $u_z(x)$ is also symmetric around the preferred policy z, i.e. if $u_z(x) = u(|z - x|)$, then $z^* = \frac{a+b}{2}$.

Assumption 2(i) is standard. Assumption 2(ii) is more restrictive but guarantees the tractability of the model, as the first order conditions imply optimality. A natural candidate for $P(v_A, v_B)$ is a *Tullock contest success function* $\frac{{v_A}^\gamma}{{v_A}^\gamma + {v_B}^\gamma}$ with noise parameter $\gamma > 0$. Note that, in this case, Assumption 2(ii) is satisfied only for $\gamma \leq 1$, although in general an optimal solution exists also if $\gamma > 1$.[4]

In terms of interpretation, the uncertainty can arise at the legislative stage, because the probability of passing either policy depends on the number of votes obtained by each candidate. Or it can arise at the voting stage, because *targeted* votes do not map perfectly into *effective* votes. As an example of a stochastic voting stage, assume that for targeted votes v_A and v_B, the effective votes for the two candidates are realizations of the random variables

$$\frac{v_A + v_B}{\theta_A v_A + \theta_B v_B}\theta_A v_A \quad , \quad \frac{v_A + v_B}{\theta_A v_A + \theta_B v_B}\theta_B v_B \tag{1}$$

for candidate A and B respectively, as a function of two shocks θ_A, θ_B iid $\sim \exp(\lambda)$. In this case, $P(v_A, v_B)$ is the probability that the effective number of votes for A is greater than the effective number of votes for B, which is given by a contest success function with $\gamma = 1$. Indeed

$$P(\theta_A v_A > \theta_B v_B) = \frac{v_A}{v_A + v_B}$$

since the shocks are independent and exponentially distributed.[5] The main results in section 3 require only Assumptions 1 and 2, while I will use the contest success function with $\gamma = 1$ as an example to derive closed-form solutions.

Finally, each citizen has a positive cost of voting. The standard assumption about voting costs in the rule-utilitarian model is that they are drawn independently from a uniform distribution $c \sim \mathcal{U}[0, \bar{c}]$. Citizens decide their participation behavior before learning the realizations of their costs. The turnout rule in each group is thus given by a threshold cost: supporters vote if their realization is below the threshold and abstain if it is above. As such, the probability of voting for a supporter corresponds to the cumulative distribution of the voting cost evaluated at the threshold. Because results change in interesting ways, my analysis will compare the case of heterogeneous voting costs with the simpler one of

[4]Herrera, Morelli and Nunnari (2016) and Bouton, Castanheira and Drazen (2020) also model uncertainty in elections via a contest success function.

[5]See Konrad (2007) for a proof. Note that the term $\frac{v_A + v_B}{\theta_A v_A + \theta_B v_B}$ in (1) serves only a normalization purpose, since the votes v_A and v_B take values in closed intervals while the shocks θ_A and θ_B can assume any positive value. Note also that the expected values of the effective number of votes are v_A and v_B for any value of the exponential distribution's parameter λ, and that the total number of votes is deterministically equal to $v_A + v_B$.

a fixed and known cost c for all citizens. In this case, I assume that the turnout rules in the groups are given directly by the probability of voting functions $p_A(z)$ and $p_B(z)$. When costs are heterogeneous, they will also be independent of policy preferences. But in a spatial framework, a threshold cost shall be allowed to vary as a function of the supporters' preferences. Hence, in this case, I will consider threshold cost functions $c_A(z)$ and $c_B(z)$ in the two groups, respectively.[6]

2.1 Ethical Reasoning

Kantian economics builds on the idea that ethical agents consider the hypothetical case in which the others behave in the same way as themselves. Agents' morality is thus founded on a principle of universalization. Specifically, in the presence of heterogeneity among agents, I follow the operationalization of such a principle by Roemer (2010, 2015, 2019), as developed in his theory of Kantian optimization. In Roemer's framework, the universalization concerns collective deviations from a prescribed rule of behavior. That is, each agent evaluates a possible deviation from a given rule by the consequences that would result if the other agents deviated similarly. The key element is what a similar way of deviating means: in the model, deviations are represented by a multiplicative factor and a similar deviation is one by the same factor. In this respect, multiplicative Kantian optimization embeds the fairness principle of proportionality in agents' moral reasoning: citizens who consider free-riding by reducing their likelihood of voting envision the other citizens also reducing their likelihood of voting by the same proportion.[7]

A second important aspect concerns who the *others* are in agents' reasoning. In the model, the universalization process applies within groups of supporters, while the voting behavior in the opposite group is taken as given, as in Nash optimization. This captures an attitude of cooperation which extends only among like-minded agents, i.e. those who share the preference for a candidate and have a common interest in solving the collective action problem associated with the turnout decision. In line with Roemer's terminology, I call the solution concept a Nash-Kantian equilibrium. The equilibrium turnout rules are thus determined by the absence of collective scalar deviations that are profitable for any member in each group, given the voting behavior in the opposite group.

[6] Considering different distributions from the uniform does not yield, instead, additional insights.

[7] Roemer also studies additive deviations, combinations between multiplicative and additive deviations, and a simpler notion of Kantian Equilibrium that applies in symmetric frameworks of identical agents, for which the universalization concerns actions and not deviations.

A third technical point concerns the fact that agents have compact strategy sets. Consider the case of a fixed and identical voting cost c for all citizens. In this case, the turnout rules in the groups are given by the probability of voting functions, $p_A(z)$ and $p_B(z)$. A multiplicative deviation σ applied to all members in group A changes the probability of voting function to $\sigma p_A(z)$. To ensure that all probabilities remain lower than 1, I assume that, for a deviation factor σ, all members z in group A deviate by the $\min\{\sigma, \frac{1}{p_A(z)}\}$. Moreover, since different members (might) vote with different probabilities, I assume that each supporter z, when evaluating potential deviations, considers only deviation factors that are bounded above by $\frac{1}{p_A(z)}$.[8] Let me then state a precise definition of the equilibrium concept; an equivalent one holds, after minimal adjustments, for the case of heterogeneous voting costs.

Definition. *A Nash-Kantian equilibrium is a pair of probability functions* $p_A(z), p_B(z)$ *such that for each group* $X \in \{A, B\}$ *no supporter* $z \in \mathcal{Z}_X$ *would prefer all supporters* $z' \in \mathcal{Z}_X$ *to vote with probability* $\min\{\sigma p_X(z'), 1\}$ *for any deviation factor* $\sigma \in [0, \frac{1}{p_X(z)}]$, $\sigma \neq 1$, *given the voting behavior in the opposite group.*
Denoting U_z *a supporter's expected utility, the equilibrium can be expressed concisely by*

$$\forall z \in \mathcal{Z}_A \quad \arg \max_{\sigma \in [0, \frac{1}{p_A(z)}]} U_z \left(\min\{\sigma p_A(\cdot), 1\}, p_B(\cdot)\right) = 1$$

$$\forall z \in \mathcal{Z}_B \quad \arg \max_{\sigma \in [0, \frac{1}{p_B(z)}]} U_z \left(p_A(\cdot), \min\{\sigma p_B(\cdot), 1\}\right) = 1$$

Note that the analytical conditions operationalize the absence of deviations by imposing that the argument σ of the maximization problem be equal to one. As a final remark, note also that the pair of probability functions $p_A(z) = 0$, $p_B(z) = 0$ for all supporters in both groups is always a Nash-Kantian equilibrium, as multiplicative deviations are in this case ineffective. In the following sections, uniqueness of the equilibrium will be claimed by restricting the analysis to strictly positive probability functions.

[8] As the next section shows, the analysis is the same without these technical assumptions; they are only needed for consistency. A voter who is voting with a high probability could otherwise propose a high deviation factor, which would be less costly for herself than for those who vote with lower probability, because of the bound at 1. This approach follows Roemer's (2010) generalized definition of the equilibrium concept for compact strategy sets.

3 Equilibrium analysis

3.1 Fixed cost of voting

Consider first the analysis in the presence of a fixed cost of voting c, given the previous definition of a Nash-Kantian equilibrium. The expected utility of a supporter z in group A is equal to

$$P(v_A, v_B)u_z(a) + (1 - P(v_A, v_B))u_z(b) - c\, p_A(z) \tag{2}$$

In equilibrium z would not want all supporters $z' \in \mathcal{Z}_A$ to deviate from their voting probability $p_A(z')$ by any factor $\sigma \in [0, \frac{1}{p_A(z)}]$. For technical convenience, we can neglect the fact that other voters z' would deviate by $\min\{\sigma, \frac{1}{p_A(z')}\}$ and work out the analysis assuming that all voters would deviate by σ, i.e. as if voters could vote with probability higher than 1. Intuitively, the benefit of a deviation by a factor σ is greater for a voter z when the constraint given by $\min\{\sigma, \frac{1}{p_A(z')}\}$ is not considered; hence if such a deviation is not profitable, it will not be when the probability of voting is bounded by 1. If a deviation by σ is followed by all voters in $\mathcal{Z}_A$, as $\int_{\mathcal{Z}_A} \sigma p_A(z) f(z) dz = \sigma v_A$, the expected utility of supporter z as a function of the deviation factor σ is equal to

$$P(\sigma v_A, v_B)u_z(a) + (1 - P(\sigma v_A, v_B))u_z(b) - c\, \sigma p_A(z) \tag{3}$$

The solution concept requires the expression in (3) to be maximized at $\sigma = 1$. Note that, since $P(\sigma v_A, v_B)$ is concave in σv_A, the previous expression is concave in σ, hence the optimality condition is given by the first order condition evaluated at $\sigma = 1$, that is

$$\frac{\partial}{\partial v_A} P(v_A, v_B) \cdot v_A[u_z(a) - u_z(b)] - p_A(z)c = 0 \tag{4}$$

Equation (4) does not pin down the function $p_A(z)$ directly, as v_A also depends on it. However, since v_A is a definite integral over $\mathcal{Z}_A$, the equation implies that $p_A(z)$ has to be proportional to the utility differential $u_z(a) - u_z(b)$, with the coefficient of proportionality to be determined in equilibrium. A similar analysis for any supporter in group B yields analogous results due to the symmetry of the framework. That is, in equilibrium we must have

$$\begin{aligned} p_A(z) &= \pi_A[u_z(a) - u_z(b)] \\ p_B(z) &= \pi_B[u_z(b) - u_z(a)] \end{aligned} \tag{5}$$

By aggregation, the numbers of votes are then equal to

$$v_A = \pi_A u_{\mathcal{Z}_A}$$
$$v_B = \pi_B u_{\mathcal{Z}_B} \tag{6}$$

where the terms $u_{\mathcal{Z}_A}$ and $u_{\mathcal{Z}_B}$ denote the aggregate utility differentials in group A and B, respectively, i.e.

$$u_{\mathcal{Z}_A} := \int_{\mathcal{Z}_A} [u_z(a) - u_z(b)] f(z) dz$$
$$u_{\mathcal{Z}_B} := \int_{\mathcal{Z}_B} [u_z(b) - u_z(a)] f(z) dz \tag{7}$$

Finally, the equilibrium values of the proportionality coefficients π_A and π_B are obtained by substituting (5) and (6) into (4) and the equivalent first order condition for group B, which can be rewritten together as

$$\frac{\partial}{\partial v_A} P(v_A, v_B) \cdot u_{\mathcal{Z}_A} = c$$
$$-\frac{\partial}{\partial v_B} P(v_A, v_B) \cdot u_{\mathcal{Z}_B} = c \tag{8}$$

A solution for the coefficients of proportionality exists.

Proposition 1. *If voting costs are fixed and identical, there exists a (strictly positive) Nash-Kantian equilibrium in which the probability of voting of each supporter in both groups is proportional to the utility differential from the candidates' policies, as given by* (5).

The existence of an interior equilibrium is guaranteed, for c within an appropriate range of values, by the Poincaré-Miranda theorem, as shown in the Appendix. If c is too low or too high, the turnout rate is either 0 or 1 in one or both groups.[9] Uniqueness of the (strictly positive) equilibrium holds in the example presented below in the section, although a proof would require specific assumptions on the shape of the function $P(v_A, v_B)$.

Proposition 1 shows that Kantian supporters assign a higher probability of voting to members with more intense preferences, as measured by the utility differential. This result recovers a dependence of voters' behavior on the utility differential from candidates' policies, which is at the core of the spatial theory of voting. It is, indeed, consistent with the observed higher likelihood of abstention by citizens who feel equally close to the competing

[9] While I omit a characterization of these cases, an equilibrium always exists because v_A and v_B take values in convex and compact sets and the payoff functions are continuous and concave by Assumption 2(ii).

candidates or too far from all of them (Enelow and Hinich 1984, Zipp 1985, Plane and Gershtenson 2004, Adams, Dow and Merrill 2006). Specifically, the shape of the function $u_z(x)$ determines how the intensity of support relates to the distance between supporters and candidates, and can thus account for the two associated notions of abstention due to indifference or alienation (Grillo 2021). In the standard spatial theory of voting, however, this relation emerges in a framework of instrumental voting, in which the utility differential is discounted by the probability of being pivotal. The Kantian approach endogenizes it in a model of ethical participation that overcomes the issue of pivotality.

Importantly, this relation would not instead follow from a simple extension of the rule-utilitarian logic within heterogeneous groups. Consider, indeed, the behavior of group rule-utilitarian supporters à la Coate and Conlin (2004), who set the turnout rule in order to maximize the aggregate utility in their group. Recalling that $v_A = \int_{\mathcal{Z}_A} p_A(z) f(z) dz$, aggregating the utilities in (2) yields a group utility equal to

$$P(v_A, v_B) \int_{\mathcal{Z}_A} u_z(a) f(z) dz + (1 - P(v_A, v_B)) \int_{\mathcal{Z}_A} u_z(b) f(z) dz - c\, v_A \tag{9}$$

Note that the aggregate utility depends on the function $p_A(z)$ only indirectly through the number of votes v_A. It follows that the maximization of group utility can determine only an optimal v_A: insofar as the optimal v_A is an interior solution, there exists an infinity of different functions $p_A(z)$ that are consistent with it. That is, the rule-utilitarian calculus pins down only the aggregate number of votes, but not the individual probabilities of voting given by $p_A(z)$. Indeed, for a given number of votes, how supporters share the voting costs through their probability of voting is irrelevant from a utilitarian perspective, since costs are identical.[10]

However, an important link between the Kantian and the rule-utilitarian models emerges from the previous analysis. Indeed, the first equation in (8) coincides with the first order condition with respect to v_A from the maximization of group utility in (9) . The same holds in group B, which implies that the solutions for the aggregate number of votes v_A and v_B are the same in the two models.

Proposition 2. *In the presence of fixed and identical voting costs, the aggregate numbers of votes v_A and v_B at the Nash-Kantian equilibrium correspond to the solutions of a group rule-utilitarian calculus.*

[10] The same is true in the framework of Feddersen and Sandroni (2006), in which the costs of voting of the whole electorate are considered.

That is, even if Kantian agents do not commit themselves ex-ante to maximizing the aggregate utility, they do maximize it in equilibrium. In a sense, under the assumption of fixed and identical voting costs, Kantian optimization can be interpreted as complementary to the group rule-utilitarian model, in that it specifies how heterogeneous supporters can share the burden of voting in order to maximize group utility.

As an example, let me then calculate the Nash-Kantian equilibrium for the case of $P(v_A, v_B) = \frac{v_A}{v_A+v_B}$ introduced in section 2, i.e. a Tullock contest success function with noise parameter $\gamma = 1$. We can readily obtain the following unique (strictly positive) solutions for the probability of voting in the two groups

$$p_A(z) = \frac{u_{\mathcal{Z}_A} u_{\mathcal{Z}_B}}{c(u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B})^2}[u_z(a) - u_z(b)]$$
$$p_B(z) = \frac{u_{\mathcal{Z}_A} u_{\mathcal{Z}_B}}{c(u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B})^2}[u_z(b) - u_z(a)]$$

By aggregation, the numbers of votes v_A and v_B are equal to

$$v_A = \frac{{u_{\mathcal{Z}_A}}^2 u_{\mathcal{Z}_B}}{c(u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B})^2} \quad , \quad v_B = \frac{u_{\mathcal{Z}_A} {u_{\mathcal{Z}_B}}^2}{c(u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B})^2} \tag{10}$$

Hence, for a Tullock contest success function, the coefficient of proportionality is the same for all supporters in both groups.[11] The coefficient of proportionality is decreasing in the cost of voting and first increasing and then decreasing in both groups' aggregate utility differentials defined in (7). The numbers of votes v_A, v_B are decreasing in the cost of voting, increasing in the aggregate utility differential of the own group, and first increasing and then decreasing in the aggregate utility differential of the opposite group. These comparative statics results offer additional insights with respect to the standard ethical voter model. The spatial framework, indeed, allows for a richer analysis of participation behavior, as a function of candidates' proposed policies a and b, voters' policy preferences $u_z(x)$, and their distribution on the policy space $f(z)$. These elements are all captured by the aggregate utility differentials $u_{\mathcal{Z}_A}$ and $u_{\mathcal{Z}_B}$.

[11]Note that, being probability functions, $p_A(z)$ and $p_B(z)$ should not be greater than 1, but to this end it suffices to assume that the voting cost c is big enough.

3.2 Heterogeneous costs of voting

Consider now the case of heterogeneous costs of voting, iid drawn from a uniform distribution, $c \sim \mathcal{U}[0, \bar{c}]$. With heterogeneous costs, a turnout rule in group A is given by a threshold cost function $c_A(z)$. The probability of voting for a supporter z is then given by the cumulative distribution function of the voting cost evaluated at the threshold, which corresponds to $p_A(z) = \frac{1}{\bar{c}} c_A(z)$. The expected voting cost is $\int_0^{c_A(z)} \frac{1}{\bar{c}} c \, dc = \frac{1}{2\bar{c}} c_A(z)^2$ and the supporter's expected utility is thus equal to

$$P(v_A, v_B) u_z(a) + (1 - P(v_A, v_B)) u_z(b) - \frac{1}{2\bar{c}} c_A(z)^2$$

Given $v_A = \int_{\mathcal{Z}_A} \frac{1}{\bar{c}} c_A(z) f(z) dz$, it still holds that, for a multiplicative deviation factor σ applied to the threshold cost, the number of votes v_A scales up by σ. When a deviation by σ is followed by all voters in group A, the expected utility of supporter z is then equal to

$$P(\sigma v_A, v_B) u_z(a) + (1 - P(\sigma v_A, v_B)) u_z(b) - \frac{1}{2\bar{c}} (\sigma c_A(z))^2$$

which is again concave in σ. Taking the first order condition and imposing $\sigma = 1$ yields

$$\frac{\partial}{\partial v_A} P(v_A, v_B) \cdot v_A [u_z(a) - u_z(b)] - \frac{1}{\bar{c}} c_A(z)^2 = 0 \tag{11}$$

By comparing the previous expression with the one in (4), we see that under uniformly distributed costs, the threshold cost must now be proportional to the square root of the utility differential. An equivalent calculation concerns group B. Given $p_A(z) = \frac{1}{\bar{c}} c_A(z)$ and $p_B(z) = \frac{1}{\bar{c}} c_B(z)$, we thus have

$$\begin{aligned} p_A(z) &= \tilde{\pi}_A \sqrt{[u_z(a) - u_z(b)]} \\ p_B(z) &= \tilde{\pi}_B \sqrt{[u_z(b) - u_z(a)]} \end{aligned} \tag{12}$$

and by aggregation

$$\begin{aligned} v_A &= \tilde{\pi}_A \tilde{u}_{\mathcal{Z}_A} \\ v_B &= \tilde{\pi}_B \tilde{u}_{\mathcal{Z}_B} \end{aligned}$$

where $\tilde{u}_{\mathcal{Z}_A} := \int_{\mathcal{Z}_A} \sqrt{[u_z(a) - u_z(b)]} f(z) dz$ and $\tilde{u}_{\mathcal{Z}_B} := \int_{\mathcal{Z}_B} \sqrt{[u_z(b) - u_z(a)]} f(z) dz$ denote the 'adjusted' aggregate utility differentials in which the square root is taken within the integral. Clearly, the probability of voting in (12) is still an increasing function of the utility differential. The proportionality coefficients $\tilde{\pi}_A, \tilde{\pi}_B$ are determined by solving (11) and the equivalent first order condition in group B, which can be rewritten together as

$$\begin{aligned} \frac{\partial}{\partial v_A} P(v_A, v_B) \cdot \frac{\tilde{u}_{\mathcal{Z}_A}}{\tilde{\pi}_A} &= \bar{c} \\ -\frac{\partial}{\partial v_B} P(v_A, v_B) \cdot \frac{\tilde{u}_{\mathcal{Z}_B}}{\tilde{\pi}_B} &= \bar{c} \end{aligned} \tag{13}$$

Proposition 3. *In the presence of heterogeneous voting costs $c \sim \mathcal{U}[0, \bar{c}]$, there exists a (strictly positive) Nash-Kantian equilibrium in which the probability of voting is an increasing function (proportional to the square root) of the utility differential from the candidates' policies, as given by* (12).

As before, existence of an interior equilibrium is guaranteed by the Poincaré-Miranda theorem, with an analogous proof as the one for Proposition 1.

The relation with the rule-utilitarian approach, however, is now different from the case of fixed and identical voting costs. First, because in this case the rule-utilitarian calculus gives a specific prediction at the individual level, which is in contrast with the one from the Kantian model and with the empirical evidence: the turnout rule maximizing aggregate utility is necessarily constant among supporters, and thus independent of the policy preferences z. Second, because the equivalence result concerning the number of votes in the aggregate does not hold. To compare the two approaches, note that the aggregate utility of group A is now given by

$$P(v_A, v_B) \int_{\mathcal{Z}_A} u_z(a) f(z) dz + (1 - P(v_A, v_B)) \int_{\mathcal{Z}_A} u_z(b) f(z) dz - \frac{1}{2\bar{c}} \int_{\mathcal{Z}_A} c_A(z)^2 f(z) dz \tag{14}$$

We have then the following result.

Proposition 4. *Consider a group rule-utilitarian calculus in the presence of heterogeneous voting costs. If there exists $c_A(z)$ such that group A utility in* (14) *is maximized, then $c_A(z) = k$. Hence, the probability of voting is the same for all supporters in a group.*

The proof of the result is provided in the appendix. To grasp the intuition, consider the case of a non-constant $c_A(z)$. Clearly, equalizing the threshold cost across different supporters z while keeping the same aggregate number of votes v_A lowers the expected costs, since group members voting with high costs are substituted by members with low costs. Hence, the rule-utilitarian prediction is stark: supporters should ignore their idiosyncratic preferences in order to make sure that aggregate costs are minimized. The culprit is the utilitarian component of the approach: what matters for group utility is only the number of votes v_A and the aggregate costs. This result under heterogeneous voting costs strengthens the advantage of the Kantian approach for a spatial analysis of voter turnout, in light of its robust prediction of heterogeneous participation rates as a function of the intensity of support.

To see why the equivalence result for the number of votes does not hold, define then the size of the two groups as $|\mathcal{Z}_A| = \int_{\mathcal{Z}_A} f(z)dz$ and $|\mathcal{Z}_B| = \int_{\mathcal{Z}_B} f(z)dz$, i.e. the shares of population belonging to each group. For a given targeted number of votes v_A in group A, rule-utilitarian members would thus set a constant threshold equal to $c_A = \frac{\bar{c}}{|\mathcal{Z}_A|} v_A$. We can then substitute this voting rule into the last term in (14) and proceed similarly for group B to obtain the following pair of first order conditions for the rule-utilitarian calculus

$$\begin{aligned} \frac{\partial}{\partial v_A} P(v_A, v_B) \cdot \frac{u_{\mathcal{Z}_A} |\mathcal{Z}_A|}{v_A} &= \bar{c} \\ -\frac{\partial}{\partial v_B} P(v_A, v_B) \cdot \frac{u_{\mathcal{Z}_B} |\mathcal{Z}_B|}{v_B} &= \bar{c} \end{aligned} \tag{15}$$

Technically, the difference between (13) and (15) arises because in the Kantian model the voting probabilities are aggregated after taking the square root of the utility differential, while the aggregation occurs before in the rule-utilitarian model. The solution is not invariant to the order of the two operations. More importantly, the result raises the question of which social welfare function Kantian optimization is implementing in the presence of heterogeneous costs, if not a utilitarian one within each group. I see this as an important and open issue for the Kantian approach, which could provide a foundation for replacing the utilitarian component of the rule-utilitarian approach with a different objective when ex-ante heterogeneity is important.

Let me turn to the example of a Tullock contest success function $P(v_A, v_B) = \frac{v_A}{v_A + v_B}$. In this case, the unique positive Nash-Kantian equilibrium is given by

$$\begin{aligned} p_A(z) &= \frac{\sqrt{\tilde{u}_{\mathcal{Z}_A} \tilde{u}_{\mathcal{Z}_B}}}{\sqrt{\bar{c}}(\tilde{u}_{\mathcal{Z}_A} + \tilde{u}_{\mathcal{Z}_B})} \sqrt{[u_z(a) - u_z(b)]} \\ p_B(z) &= \frac{\sqrt{\tilde{u}_{\mathcal{Z}_A} \tilde{u}_{\mathcal{Z}_B}}}{\sqrt{\bar{c}}(\tilde{u}_{\mathcal{Z}_A} + \tilde{u}_{\mathcal{Z}_B})} \sqrt{[u_z(b) - u_z(a)]} \end{aligned}$$

The comparative statics properties are the same as in the case of fixed and identical voting costs. The coefficient of proportionality is the same for all supporters in both groups, it is decreasing in the cost of voting and first increasing and then decreasing in the aggregate utility differentials of both groups. The aggregate numbers of votes are in this case

$$v_A = \frac{\tilde{u}_{\mathcal{Z}_A}\sqrt{\tilde{u}_{\mathcal{Z}_A}\tilde{u}_{\mathcal{Z}_B}}}{\sqrt{\bar{c}}(\tilde{u}_{\mathcal{Z}_A}+\tilde{u}_{\mathcal{Z}_B})} \quad , \quad v_B = \frac{\tilde{u}_{\mathcal{Z}_B}\sqrt{\tilde{u}_{\mathcal{Z}_A}\tilde{u}_{\mathcal{Z}_B}}}{\sqrt{\bar{c}}(\tilde{u}_{\mathcal{Z}_A}+\tilde{u}_{\mathcal{Z}_B})}$$

which are decreasing in the cost of voting, increasing in the aggregate utility differential of the own group, and first increasing and then decreasing in the aggregate utility differential of the opposite group.[12]

4 Discussion

I address here a few relevant issues, before concluding. I first discuss the endogenization of candidates' policies a and b. I then examine whether one could amend the rule-utilitarian approach, instead of abandoning it, in order to improve its predictions at the individual level. Finally, I offer a critical perspective on the *Kantian* label that economists typically assign to a reasoning based on a universalization principle.

4.1 Candidates' choice of policies

In order to focus on citizens' turnout behavior, I have taken as given candidates' policies a and b. The advantage of a spatial framework, however, is also to study how candidates choose their platforms on the policy space. While determining policies endogenously is beyond the scope of this paper, I sketch here some preliminary considerations. The analysis of policy choices requires assumptions on candidates' objective. In the classical framework by Downs (1957), candidates are purely office-motivated and therefore maximize their probability of winning. In the model of this paper, office-motivation corresponds to maximizing $P(v_A, v_B)$ for candidate A and to minimizing it for candidate B. Note that this implies

[12] The rule-utilitarian solutions are in this case

$$v_A = \frac{(u_{\mathcal{Z}_A}|\mathcal{Z}_A|)^{\frac{3}{4}}(u_{\mathcal{Z}_B}|\mathcal{Z}_B|)^{\frac{1}{4}}}{\bar{c}(\sqrt{u_{\mathcal{Z}_A}|\mathcal{Z}_A|}+\sqrt{u_{\mathcal{Z}_B}|\mathcal{Z}_B|})} \quad , \quad v_B = \frac{(u_{\mathcal{Z}_A}|\mathcal{Z}_A|)^{\frac{1}{4}}(u_{\mathcal{Z}_B}|\mathcal{Z}_B|)^{\frac{3}{4}}}{\bar{c}(\sqrt{u_{\mathcal{Z}_A}|\mathcal{Z}_A|}+\sqrt{u_{\mathcal{Z}_B}|\mathcal{Z}_B|})}$$

which show the same comparative statics properties with respect to the cost of voting and the aggregate utility differentials. The relative magnitude of v_A, v_B in the two models depend on $\bar{c}, a, b$ as well as on the shape of $u_z(\cdot)$.

that candidates care only about the numbers of votes v_A and v_B, and not about the specific distribution of individual voting probabilities given by $p_A(z)$ and $p_B(z)$.

In my analysis, independently of citizens' ethical calculus and in both cases of fixed and heterogeneous voting costs, the aggregate numbers of votes v_A and v_B depend crucially on the groups' aggregate utility differentials. Consider the example of a Tullock contest function $P(v_A, v_B) = \frac{v_A}{v_A+v_B}$ in the case of a fixed cost of voting, whose solutions for the number of votes are given in (10). It is easy to check that candidates' maximization problems correspond to

$$\max_a \frac{u_{\mathcal{Z}_A}}{u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B}} \quad , \quad \max_b \frac{u_{\mathcal{Z}_B}}{u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B}}$$

The aggregate utility differentials $u_{\mathcal{Z}_A}$ and $u_{\mathcal{Z}_B}$, in turn, depend on the primitive elements of the model, such as citizens' distribution on the policy space $f(z)$ or the shape of their utility function $u_z(x)$. In general, thus, also the existence and properties of the political equilibrium for candidates' policies a and b depend on such primitive elements. In a related paper (Grillo 2021), I examine in more detail candidates' strategies and provide conditions on $f(z)$ and $u_z(x)$ for a result of turnout-driven polarization, occurring when candidates pursue an electoral strategy of mobilization.

4.2 More on the comparison between ethical models

The Kantian framework overcomes the shortcomings of the utilitarian aggregation rule by resorting to a different ethical foundation. Strong defendants of the utilitarian principle, however, could argue for a smaller amendment of the rule-utilitarian model, which simply reduces the level at which supporters' utility is aggregated. Consider for example a simpler model in which each group is further divided in two subgroups: a subgroup supports the candidate strongly, while the other only weakly. In this case, *subgroup* rule-utilitarian agents would maximize the aggregate subgroup utility taking as given the voting behavior both in the other subgroup of supporters of the same candidate and in the two subgroups of supporters of the opposing candidate. In the presence of heterogeneous costs, then, the logic behind Proposition 4 would make the probability of voting constant among members of a subgroup but would not prevent different threshold costs between subgroups. In this case, strong supporters could vote with a higher probability than weak supporters, showing a positive relationship between participation and the intensity of support.

There are, however, two inconveniences of a *subgroup* rule-utilitarian model. The first is the analytical complexity, as already in my simple example the strategic interaction involves four different maximization problems. The second is that when the reduction of the level of aggregation is taken to the limit, one stumbles back onto the issue of pivotality. If the distribution of citizens over the policy space is continuous, as in my model, every set of citizens sharing the same preferred policy has mass zero. Hence, one cannot take the level of aggregation down to voters' preferred policy, because the aggregate utility would then coincide with the individual utility, and supporters within a subgroup would even collectively be unable to affect the outcome of the election. Kantian optimization, instead, provides a tractable model to account for voters' idiosyncratic preferences, even in the presence of a continuous distribution.

4.3 On the ***Kantian*** label

A reference to Kant is customary in economics to denote the counterfactual reasoning of agents who envision a universalization of their behavior. With respect to the ethical voting model, it is interesting that an analogous mention of Kant is made by Feddersen (2004) in justifying the rule-utilitarian approach, which follows Harsanyi's (1977) tribute to Kant's intellectual tradition of claiming a requirement of universality for moral rules. Indeed, while the universalization principle is explicit in Roemer's Kantian optimization, it is somehow also implicit in the rule-utilitarian calculus, as the prescribed rule is optimal only if followed by everyone.

It is fair to note, however, that the Kantian label may also generate some misunderstanding, as Kant's moral philosophy is generally understood as fully non-consequentialist. The Kantian approach in economics is also non-consequentialist in that agents consider a hypothetical scenario rather than the true consequences of their strategy choices, but it builds on a consequentialist interpretation of the universalization principle, which is more in line with rule-consequentialism than with Kant's view of the universalization as a test for contradictions. In my opinion, one could alternatively describe Kantian optimization as a model of egoistic (non-utilitarian) rule-consequentialist agents.[13] A valuable discussion on Kantian rationality is in Sugden (1991), while a critique of the Kantian label has been expressed by Wolfelsperger (1999) and Ballet and Jolivet (2003). As a comparison, White

[13] Roemer (2019, Ch.1) acknowledges using the term Kantian for its suggestive meaning and not to imply a deeper Kantian justification.

(2004, 2019) discusses how the paradigm of homo economicus can relate to Kantian moral philosophy in its orthodox deontological interpretation.

5 Conclusion

I have studied electoral participation within heterogeneous groups, made of supporters who share a preference for a candidate but have different intensities of support. In a spatial setting, the difference in intensities comes from voters' underlying preferences on a policy dimension. In this framework, Kantian behavior à la Roemer yields substantial participation and predicts the likelihood of voting as an increasing function of the intensity of preferences. The result is consistent with the theoretical and empirical literature showing that voters' distance from candidates affect their likelihood of participating. By contrast, the rule-utilitarian logic, despite yielding accurate comparative statics properties in the aggregate, fails to account for the heterogeneity at the individual level. If voting costs are identical among supporters, however, the number of votes coincide in the two frameworks. Hence, in this case, the Kantian approach can be complementary to the rule-utilitarian one by specifying a rule at the individual level on which supporters can coordinate.

Kantian optimization delivers finer predictions at the individual level because agents maximize an individual utility, which accounts for their idiosyncratic preferences. Their ethical principle consists in universalizing, within their group, their possible deviations from a turnout rule. With respect to Roemer's broader approach, I have only considered deviations in a multiplicative form. In this respect, in light of the evidence that moral judgements are often consistent with the universalization logic (Levine et al. 2020), further research is needed to corroborate the positive content of multiplicative deviations. From a theoretical standpoint, considering the case in which the universalization does not imply the same scalar deviation for all concerned agents but allows for heterogeneity in deviations also represents a promising direction for extending the framework. Finally, a better understanding of how Kantian morality relates to welfare aggregation, and specifically of which social welfare function it implements under different modeling assumptions, is an important question that is open for investigation.

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A Appendix

Proof of Proposition 1:

Rewrite system (8) as

$$\frac{\partial}{\partial v_A} P(v_A, v_B) \cdot u_{\mathcal{Z}_A} - c = 0$$
$$-\frac{\partial}{\partial v_B} P(v_A, v_B) \cdot u_{\mathcal{Z}_B} - c = 0$$

We have $v_A \in [0, |\mathcal{Z}_A|]$ and $v_B \in [0, |\mathcal{Z}_B|]$ where $|\mathcal{Z}_A| = \int_{\mathcal{Z}_A} f(z)dz$ and $|\mathcal{Z}_B| = \int_{\mathcal{Z}_B} f(z)dz$ are the population shares of the two groups, with $|\mathcal{Z}_A| + |\mathcal{Z}_B| = 1$. By assumption 2(ii), $\frac{\partial}{\partial v_A} P(v_A, v_B)$ is decreasing in v_A and $-\frac{\partial}{\partial v_B} P(v_A, v_B)$ is decreasing in v_B. Then if c takes a value in the range satisfying

$$\max_{v_B} \frac{\partial}{\partial v_A} P(|\mathcal{Z}_A|, v_B) \cdot u_{\mathcal{Z}_A} < c < \min_{v_B} \frac{\partial}{\partial v_A} P(0, v_B) \cdot u_{\mathcal{Z}_A}$$

and

$$\max_{v_A} -\frac{\partial}{\partial v_B} P(v_A, |\mathcal{Z}_B|) \cdot u_{\mathcal{Z}_B} < c < \min_{v_A} -\frac{\partial}{\partial v_B} P(v_A, 0) \cdot u_{\mathcal{Z}_B}$$

we have $\frac{\partial}{\partial v_A} P(0, v_B) \cdot u_{\mathcal{Z}_A} - c > 0$ and $\frac{\partial}{\partial v_A} P(|\mathcal{Z}_A|, v_B) \cdot u_{\mathcal{Z}_A} - c < 0 \ \forall v_B$, and as well $-\frac{\partial}{\partial v_B} P(v_A, 0) \cdot u_{\mathcal{Z}_B} - c > 0$ and $\frac{\partial}{\partial v_B} P(v_B, |\mathcal{Z}_B|) \cdot u_{\mathcal{Z}_B} - c < 0 \ \forall v_A$. Hence, by the Poincaré-Miranda theorem, system (8) has a solution $(v_A, v_B) \in [0, |\mathcal{Z}_A|] \times [0, |\mathcal{Z}_B|]$ and thus, given (6), a solution for the proportionality coefficients π_A, π_B.

If the voting cost c is too low or too high, an equilibrium still exists but it will be a corner solution, for which turnout is 0 or 1 in one or both groups. We can indeed see system (8) as yielding the "best reply functions" for v_A and v_B: a Nash equilibrium in pure strategies then exists by the Nash-Debreu theorem, since v_A, v_B take values in compact and convex sets and payoff functions are continuous and concave by Assumption 2(ii).

The proof of Proposition 3 is analogous to the proof of Proposition 1.

Proof of Proposition 4:

Consider any non-constant continuous $c_A(z)$ and the resulting aggregate number of votes $v_A = \int_{\mathcal{Z}_A} \frac{1}{\bar{c}} c_A(z) f(z) dz$. There always exists a constant turnout rule k that yields the same number of votes v_A but lower aggregate voting costs, i.e. such that $\int_{\mathcal{Z}_A} \int_0^{c_A(z)} \frac{1}{\bar{c}} c \, dc f(z) dz > \int_{\mathcal{Z}_A} \int_0^{k} \frac{1}{\bar{c}} c \, dc f(z) dz$. Hence the aggregate group utility is higher under the voting rule k than under the voting rule $c_A(z)$. One can alternatively solve the corresponding calculus of variations problem

$$\min_{c_A(z) \in \mathcal{C}} \int_{\mathcal{Z}_A} F(z, c_A(z)) dz \quad \text{subject to} \quad \int_{\mathcal{Z}_A} G(z, c_A(z)) dz = v_A$$

where $F(z, c_A(z)) = \int_0^{c_A(z)} \frac{1}{\bar{c}} c \, dc$ and $G(z, c_A(z)) = \int_0^{c_A(z)} \frac{1}{\bar{c}} dc$. The augmented Lagrangian is $\int_{\mathcal{Z}_A} F(z, c_A(z))dz + \lambda\, G(z, c_A(z))dz$ and the Euler-Lagrange equation gives

$$\frac{\partial F}{\partial c_A(z)} + \lambda\, \frac{\partial G}{\partial c_A(z)} = \frac{(c_A(z) + \lambda)}{\bar{c}} = 0$$

from which we obtain that $c_A(z)$ is constant. Note that the second variation equals $\int_{\mathcal{Z}_A} \frac{1}{\bar{c}} v^2 dz$ which is positive definite for all variations $v(z)$, and hence the solution is indeed a minimizer.