More Ambiguous or More Complex? Investigating Model Uncertainty in Ellsberg Urns*

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November 12, 2021

Abstract

We explore experimentally individual preferences under model uncertainty using Ellsberg two-color urns of different sizes. We find that both (i) preferences for risk over ambiguity and (ii) preferences among different ambiguous situations *do* depend on the size of the urns considered. We define and use different notions of "more ambiguous" and "more complex" to build hypotheses on what drives such preferences under uncertainty. We show that disentangling the effects due to specific attitudes towards ambiguity and complexity provide an explanation of the heterogeneity observed in our data. Overall, our results highlight the importance of considering the structure of model uncertainty for comparing different uncertain situations.

Keywords: Ambiguity, model uncertainty, smooth ambiguity aversion, Ellsberg paradox

JEL Classification: D81

*This research is supported by a grant from the French Agence Nationale de la Recherche (ANR-17-CE03-0008-01 INDUCED), the Region Hauts-de-France (2021.00865 CLAM), and from the European Research Council (ERC-2013-StG 336703-RISICO). We are grateful to Mohammed Abdellaoui, Aurélien Baillon, David Crainich, Simone Cerreia-Vioglio, Louis Eeckhoudt, Thomas Epper, Itzhak Gilboa, Sujoy Mukerji, Peter Wakker and especially Massimo Marinacci for helpful comments and discussions. We thank Caterina Verrone and Diana Valerio for their help with the laboratory experiment. Logistic support from the Bocconi Experimental Laboratory in the Social Sciences for hosting our experimental sessions is kindly acknowledged.

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1 Introduction

More often than not, the ex-ante information that a decision-maker (DM) has at her disposal is not sufficient to single out a unique probability model (or distribution) to quantify the uncertainty over the relevant states of the world. Consequently, most decisions are actually made in situations of *ambiguity*, where probabilities are not (perfectly) known. In such situations, it may be convenient to decompose ambiguity into different *layers* of analysis, among which the layers of *risk* and *model uncertainty* (Hansen, 2014; Marinacci, 2015; Hansen and Marinacci, 2016; Aydogan et al., 2020). Risk characterizes the uncertainty within a given probability model and, as such, features an aleatory (or physical) type of uncertainty, which is typically represented by an objective probability measure. In contrast, model uncertainty characterizes the uncertainty across different probability models, thus featuring an epistemic type of uncertainty, which can be quantified by subjective probabilities.

Ellsberg's (1961) classical thought experiments illustrate particularly well the distinction between the layers of risk and model uncertainty. In his two-color problem, a decision maker decides to bet on a draw from one of two urns: a known urn that contains 50 red and 50 black balls, and an unknown urn that contains 100 balls, each of which is either red or black. In such a context, a risk is fully characterized by a specific composition of the urn. Accordingly, the known urn is an instance of risk only whereas model uncertainty is absent by construction. In contrast, the unknown Ellsberg urn that contains 100 balls in it features both layers of model uncertainty and risk: there is uncertainty across 101 possible compositions, each of which represents a different risk. The widespread finding of ambiguity aversion in Ellsberg-type experiments (i.e., the preference for betting on the known over the unknown urn) indicates behavioral differences towards the layers of model uncertainty and risk.[1]

In this paper, we provide an experimental investigation of preferences under model uncertainty that characterizes Ellsberg urns. Although Ellsberg two-color problem has been widely used for calibrating ambiguity attitudes in empirical studies (Trautmann and Van De Kuilen, 2015), a distinction is rarely made between urns that contains different number of balls. However, the size of an unknown urn proves critical from a theoretical point of view: it determines the characteristics of the set of models to consider, and thus, implicitly, the degrees of ambiguity and complexity of the situation. Such characteristics may, ultimately, matter in determining

[1]Various theories have been proposed to accommodate Ellsberg-type behaviors (e.g., Segal, 1987, Gilboa and Schmeidler, 1989, Ghirardato et al., 2004, Klibanoff et al., 2005, Maccheroni et al., 2006, and Seo, 2009; see also Gilboa and Marinacci, 2013, and Machina and Siniscalchi, 2014, for surveys).

the DM's preferences. Thus, in this paper, we investigate model uncertainty in Ellsberg urns by comparing Ellsberg's original ambiguous urn ($N = 100$), denoted $E100$, with two extreme cases: (i) a (very) small-sized urn, $E1$, which contains only one ball ($N = 1$), and (ii) a large-sized urn, $E1000$ made of 1000 balls ($N = 1000$). $E1$ is extreme in the sense that the probability of drawing a red (or black) ball is either 0% or 100%, with no intermediate probabilities within these bounds. $E1000$ is the other extreme, presenting a very large number (1001) of potential compositions.

We use two types of relation to build testable predictions and analyze preferences over different situations of model uncertainty. The first one is the *more ambiguous* relation, whose alternative definitions have been proposed by Jewitt and Mukerji (2017) and Izhakian (2020). Jewitt and Mukerji's (2017) definition relies on a specific class of preferences, whose elements come from a given family. In line with their study, we consider two parametric families of preferences, namely (i) the α-maxmin expected utility family and (ii) the smooth ambiguity family. For these families, the degree of ambiguity depends, for (i), on the range (i.e., maximum and minimum) of the the distribution of expected utilities induced by different sets of probability models, and, for (ii), on its spread. On the other hand, the more ambiguous relation of Izhakian (2020) relies on a measure of expected volatility in probabilities. The second relation is based on the notion of complexity. We propose two definitions of *more complex* that enable us to rank situations of model uncertainty in terms of complexity by comparing directly their sets of potential models. Our first definition equates the degree of complexity to the cardinality of the set of potential models. Our second definition is based on a partial ordering of the sets of potential models according to its coarseness (or fineness).

Our empirical results illustrate the usefulness of considering both relations of more ambiguous and more complex when comparing different situations of model uncertainty. In particular, we show that (1) the distinction between risk and Ellsberg ambiguity depends on the underlying set of models considered; (2) there exists preferences over Ellsberg urns of different sizes, with a tendency to prefer larger-sized urns; and (3) the heterogeneity in size preferences can be explained by attitudes towards the degree of ambiguity and degree of complexity of the situation.

Related investigations with Ellsberg urns have been conducted in other experiments. Among them, the recent study of Filiz-Ozbay et al. (2021) is the closest to ours. Whereas our study focus on the decomposition of risk and model uncertainty to disentangle the roles of ambiguity and complexity induced by different compositions in Ellsberg urns, Filiz-Ozbay et al. (2021) focus on preferences for the size of the ambiguous urn and analyze the role of ratio bias. Although we do not explicitly consider the ratio bias as in Filiz-Ozbay et al. (2021), we control for a potential denominator effect by comparing situations characterized by the same sets of potential models but generated by urns containing different numbers of balls. Another study close to ours is that of Chew et al. (2017). In their experiment, Chew et al. (2017) provide the subjects with either no information (a situation called full ambiguity) or partial information (a situation called partial ambiguity) regarding the possible compositions of an Ellsberg urn with 100 balls. In contrast to Chew et al. (2017), our experiment focuses on situations of full ambiguity (except when controlling for the denominator effect). Finally, using alternative designs, the studies of Armantier and Treich (2016) and Kovářík et al. (2016) highlight the role of complexity when measuring ambiguity preferences. While those studies did not propose any formal definition of complexity under ambiguity, we propose such definitions under model uncertainty.

The paper is organized as follows. Section 2 presents our experimental design. Section 3 outlines the theoretical framework we use to analyze the uncertain situations in terms of their degree of ambiguity and degree of complexity, and to build testable hypotheses. We report our main findings in Section 4. We further discuss our design and findings in relation to previous studies in Section 5. Section 6 concludes.

2 Experimental design

We use a within-subject design to study individual choices under risk and Ellsberg ambiguity. The experiment entails betting on the color of a ball drawn from an urn in different situations. All situations entail a standard two-color Ellsberg (1961) setting. The experiment used real monetary incentives.

2.1 The choice situations

The subjects in our experiment are confronted with five different uncertain situations. These situations are represented by urns containing balls that can be either red or black. They are characterised as follows:

1. Risk (denoted R): the urn contains 50 red and 50 black balls;
2. Ellsberg's ambiguity with 1 ball (denoted $E1$): the urn contains 1 ball, which can be either red or black;
3. Ellsberg's ambiguity with 100 balls (denoted $E100$): the urn contains 100 balls, each of which can either be red or black;
4. Ellsberg's ambiguity with 1000 balls (denoted $E1000$): the urn contains 1000 balls, each of which can be either red or black.

5. Partial ambiguity (denoted $P100$): the urn contains 100 balls that are either *all* red or *all* black.

The urns used in situations R and $E100$ are the same as the ones used by Ellsberg (1961) in his original two-color problem. Urns $E1$ and $E1000$ are similar, in spirit, to $E100$ but contain, respectively, less (i.e., 1) and more (i.e., 1000) balls in total. Finally, $P100$ is identical to $E1$, except for the number of balls contained in the urn.[2]

A representation of these urns is given in Figure 1.

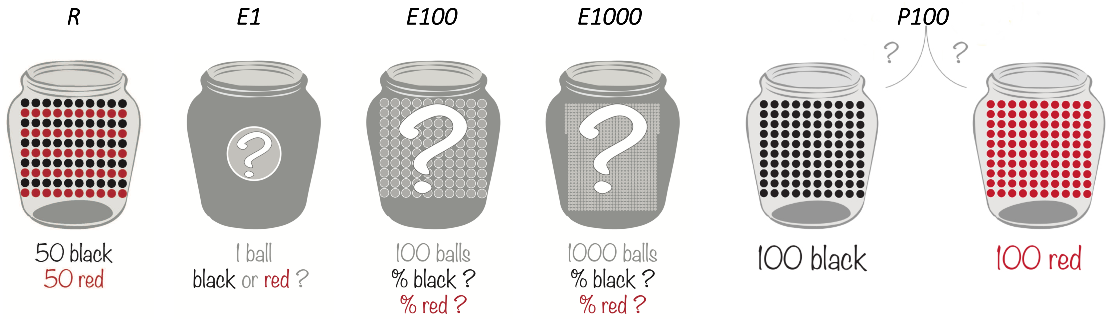

Figure 1: The Different Uncertain Situations Represented by Urns

Subjects were required to answer several questions involving bets on the color of a ball drawn from each of these urns. For each of the five uncertain situations, the subjects were given the choice of the color on which to bet and were offered €15 for a correct bet and €0 otherwise. We elicited direct preferences between betting on one urn or the other using a random lottery pairs (RLP) design (Harrison and Rutström, 2008).[3]

2.2 Procedure

The experiment was run on computers at the Bocconi Experimental Laboratory for Social Sciences where 84 Bocconi University students participated. Four sessions were organized with 19 to 24 subjects per session. Subjects were paid in cash at the end of the experiment. Average earnings were approximately €14.5, including a €5 participation fee. Each session lasted approximately one hour, including instructions and payment. The experiment started with the experimental instructions, examples of the stimuli, and related comprehension questions. Complete instructions are available in online Appendix S1.

[2] $P100$ also corresponds to one of the urns used in Chew et al. (2017) to study the notion of *partial ambiguity*, which implicitly relies on the partial information available to pin down the potential probability models describing the phenomenon of interest (see also Berger, 2021). In $P100$, the information available is that all balls have the same color.

[3] In the context of another study, we also elicited the certainty equivalents (CEs) of each bet using a price-list design. The order of the RLP and CE elicitations was randomized and no order effect was detected. For details, see Online Appendix S3.

Stimuli During the experiment, subjects faced five different uncertain situations, represented by the urns described in Section 2.1. All the urns were constructed before each session by an assistant, who was not present in the lab during the experiment. Thus, no one in the room (including the experimenters) had more information about the content of the urns than that described in the experimental instructions. The subjects were told that they would have the opportunity to look at the urns at the end of the experiment to check the truthfulness of the instructions.

We presented the different uncertain situations two-by-two in a randomized sequence and asked subjects the urn on which they prefer to bet (to win €15). After they selected one of the two urns, the same question was asked again, but this time with a slightly increased amount (€15.10) for a correct bet on the urn that was not selected at the first stage (see Filiz-Ozbay et al., 2021). Subjects were then considered as strictly preferring one of the two urns if they chose it in both stages and as indifferent if they reversed their choice in the second stage.[4] In total, nine of the ten possible binary choices over the five urns were presented to the subjects, resulting in eighteen choice questions.[5] Finally, at the end of the experiment (and before the payment stage), the subjects answered a short survey with a few socio-economic questions.

Payment and incentives Each subject received a €5 flat payment for taking part to the experiment. In addition, they were paid depending on one of the decisions they made in the experiment. A prior random incentive system was implemented to determine the choice question that was used for determining the subjects' payment at the end of the experiment.[6] After all the subjects answered all the questions, a ball was drawn from the urn corresponding to the relevant choice question and the subjects' decision in that question was observed. Each subject was then paid the amount corresponding to her decision. See online Appendix S1 for more details.

[4]Note that if a subject was indeed indifferent between the two bets in the first place, any $\varepsilon > 0$ prize increase would lead to a reversal in the second stage. Although we cannot rule out, theoretically, the possibility of a strict, but low, preference for the urn initially chosen, which is then reversed by the additional prize in the second stage, in our view, €0.10 the additional prize we used is sufficiently small to distinguish strict preference from indifference.

[5]The binary choice between $P100$ and $E1000$ was not included as the preference between them was not one of our main interests. See details in Section 3.

[6]Under this random incentive system, the randomization is performed before subjects begin answering questions (Johnson et al., 2020). Such a *prior* incentive system aims to enhance isolation to minimize potential biases, thereby preventing subjects from hedging over the randomization between problems (see Baillon et al., 2014; for a demonstration of its incentive compatibility in Ellsberg-type experiments, and Epstein and Halevy, 2019; for a recent application).

3 Theoretical considerations

In this section, we outline different approaches that have been proposed in the literature to differentiate ambiguous situations.

3.1 Setup

Let S denote a finite set of states of the world and C a set of consequences. Formally, an act (or bet) is a function $a : S \rightarrow C$ mapping states into consequences. The collection of all acts is denoted by A. In our case, each of the acts we consider involves a bet on the color (red or black) of a ball drawn from an urn. An act a_i results in a consequence $c \in \{€15, €0\}$ depending on which state of the world $s_i \in \{\text{red}, \text{black}\}$ is realized in $i \in \{R, E1, E100, E1000, P100\}$. The state space considered consists of 2^5 states. Yet we restrict our attention to 5 payoff-relevant events each describing whether the bet is correct or not in a given situation. We consider a DM who has a complete and transitive preference relation $\succsim$ over acts.[7] Following Wald (1950), we assume that the DM knows that states are generated by a probability model that is presumed to belong to a collection M, which is taken as a datum of the decision problem. In our setup, each model describes a possible composition of the urn. To ease the derivation of our predictions, we use the following symmetry assumption.

Symmetry: For each act a_i, the DM is indifferent to the color on which to bet (red or black).

Such a symmetry assumption is common in the ambiguity literature and has been supported empirically in various studies (e.g., Abdellaoui et al., 2011; Chew et al., 2017; Epstein and Halevy, 2019; Aydogan et al., 2020).

3.2 Decomposing ambiguity into layers

Following Hansen (2014); Marinacci (2015), and Hansen and Marinacci (2016), Ellsberg ambiguity may be decomposed into the *layers* of *risk* (uncertainty with known probabilities) and *model uncertainty* (uncertainty about which probability model should be used).[8] A risk is typically characterized by a unique, objective probability measure. For example, any known two-color urn (e.g., R) represents a

[7] As usual, $\sim$ denotes indifference and $\succ$ strict preference. In consequence, a $\succsim$ b means that the DM either strictly prefers act a to act b or is indifferent between the two.

[8] Note that, in most real-life situations, a third layer of uncertainty, known as *model misspecification* (uncertainty about whether or not the correct model lies among the set of models considered) is also present (see Aydogan et al., 2020). However, as all the situations considered in our experiment can be analyzed in terms of risk and model uncertainty only, we abstract from model misspecification issues.

risk, which may be expressed as the binary lottery x_py, yielding x with an objective probability p and y otherwise (e.g., $15_{0.5}0$ in our experiment). The uncertainty featured in the layer of risk has therefore an aleatory (or physical) nature, which is due to the intrinsic randomness that states feature. Ambiguous urns, on the contrary, are characterized by a multiplicity of possible probability distributions. In principle, it is possible to posit a set M of potential models $m_p(s)$ describing the likelihood of the different states. A second layer of uncertainty therefore arises about which is the correct model to consider among the collection $M = \{m_p\}$ (i.e., among all the the possible compositions of the urn). While each probability model $m_p \equiv x_py$ is itself a given risk, the uncertainty in this second layer can no longer be quantified objectively, and as such is said to have an epistemic nature. If one can still form a probability measure over the possible urn compositions, such probabilities are necessarily *subjective*, reflecting the degree of belief one has in each possible model.

Table 1: Characteristics of the uncertain situations

Uncertain situation	Number of balls (N)	Set of models (M)	Number of models ($\|M\|$)	Volatility of probabilities ($\mho^2$)[a]
R	100	$\left\{\frac{50}{100}\right\}$	1	0.000
$E1$	1	$\left\{\frac{0}{1}, \frac{1}{1}\right\}$	2	0.250
$E100$	100	$\left\{\frac{0}{100}, \frac{1}{100}, \ldots, \frac{100}{100}\right\}$	101	0.085
$E1000$	1000	$\left\{\frac{0}{1000}, \frac{1}{1000}, \ldots, \frac{1000}{1000}\right\}$	1001	0.0835
$P100$	100	$\left\{\frac{0}{100}, \frac{100}{100}\right\}$	2	0.250

Notes: [a] Assuming a uniform prior probability measure μ

The sets M of possible models to consider in the uncertain situations of our experiment are summarized in Table 1 (in an abuse of notations, we let each model $m_p \equiv x_py$ be fully characterized by its probability p). Notice that situations $E1$ and $P100$ share the same set of models M, but differ in their total number of balls. On the contrary, situations $E100$ and $P100$ have the same number of balls but differ in terms of their set M.

In what follows, we present two types of relation that will be used to differentiate ambiguous situations under the two-layer decomposition. These relations will allow us to make predictions on the preferences over Ellsberg urns of different sizes (i.e., over $R, E1, E100$, and $E1000$). As the sets of models considered in $E1$ and $P100$ are the same, the approaches we follow do not make any distinction between these two situations. Thus, we attitubute the behavioral difference that exists between $P100$ and $E1$ (if any) to a framing (denominator) effect, which we test in Section 4.4.

3.3 The more ambiguous relation

We now present two approaches that have recently proposed in the literature to order the different situations of our experiment by their degree of ambiguity. The first approach is based on the notion of *more ambiguous* of Jewitt and Mukerji (2017). This notion allows for establishing a partial ordering among ambiguous situations within a given class of preferences. As the more ambiguous relation is based on the notion of more ambiguity averse, it is first essential to adopt a normalization for *ambiguity neutrality.*

Subjective expected utility Following Ghirardato and Marinacci (2002), we consider subjective expected utility (SEU) as a benchmark for ambiguity neutrality.[9] Under SEU, it is assumed that the DM has a subjective prior probability measure $\mu : 2^M \to [0, 1]$ quantifying the epistemic uncertainty in the second layer of model uncertainty. The symmetry condition implies that the subjective probability distribution μ over the set of probability models is symmetric. For example, for $E1$, the symmetry condition implies that $\mu(0\%) = \mu(100\%) = 0.5$. The two-layer version of SEU that has been axiomatized by Cerreia-Vioglio et al. (2013) takes the form:

$$V_{\text{SEU}}(a_i) = \sum_{m_p \in M} \Big(\sum_{s \in S} u\left(a_i(s)\right)\ m_p(s) \Big)\, \mu(m_p). \tag{1}$$

In this expression, u is a von Neumann-Morgernstern utility function, translating economic consequences (measured in monetary terms) into utility levels. This function captures risk attitudes. Model uncertainty is then addressed using the subjective prior probability distribution μ that quantifies the DM's belief about the correct urn composition (and thus about p). Under this framework, the layers of risk and model uncertainty are implicitly treated in the same way (Marinacci, 2015). In our case, this means that all the bets on the different urns are evaluated similarly. For example, assuming a uniform prior μ, we have, in the case of a bet yielding $x = 15$ if correct and $y = 0$ otherwise and after normalizing $u(0) = 0$:

[9] The choice of SEU is a minimal assumption: this preference intuitively embodies ambiguity neutrality (even if it might not be the only one) and seems to be the most obvious one (Gilboa and Marinacci, 2013). This benchmark is moreover particularly well adapted for the ambiguity models we consider: SEU corresponds to a linear ambiguity function ϕ in the smooth ambiguity model of Klibanoff et al. (2005), and Marinacci (2002) shows that it is essentially without loss of generality to assume SEU as the benchmark model for ambiguity neutrality for α-maxmin preferences (for more details, see Jewitt and Mukerji, 2017).

$$V_{\mathrm{SEU}}(a_i) = \sum_p \Big(pu(15)\Big)\, \mu(p) = \frac{1}{2}u(15) \quad \forall i. \tag{2}$$

Thus, SEU predicts the following *indifference* pattern:

$$R \sim E1 \sim E100 \sim E1000. \tag{3}$$

Ambiguity averse preferences are then defined in relation to ambiguity neutrality. Specifically, we will consider as ambiguity averse preferences those that are more ambiguity averse than ambiguity neutrality.

The *more ambiguous* relation is defined as follows.

Definition: More Ambiguous (1) [Jewitt and Mukerji, 2017] Let $\mathcal{P}$ be a class of preferences over a set A. Assume that a binary relation "more ambiguity averse" is given, which is a strict partial order and that each $\succsim \in \mathcal{P}$ is related to an ambiguity neutral element of $\mathcal{P}$. Given two acts f, $g \in$ A, f is a *more ambiguous* act than g if the following conditions are satisfied:

(i) if $\succsim \in \mathcal{P}$ is ambiguity neutral, then $g \sim f$;

(ii) for all $\succsim_A$, $\succsim_B \in \mathcal{P}$ such that $\succsim_A$ is an ambiguity neutral preference and $\succsim_B$ is more (less) ambiguity averse than $\succsim_A$, we have $g \succsim_B (\precsim_B) f$.

According to this definition, an act f is more ambiguous than an act g if an ambiguity-averse DM prefers g to f, but an ambiguity-neutral DM is indifferent between the acts. It should be clear, however, that this order of more ambiguous arises on the back of a specific *more ambiguity averse* relation on preferences. In what follows, we analyze two distinct families of preferences, the smooth ambiguity family and the α-maxmin expected utility family.

Smooth ambiguity model Under the smooth ambiguity model proposed by Klibanoff et al. (2005), the two layers of uncertainty are also quantified using a single probability measure. However, this approach allows for a distinct treatment of the layers of risk and model uncertainty. In particular, by letting a function $\phi \equiv v \circ u^{-1}$ represent the DM's attitude towards ambiguity resulting from the composition of attitudes towards model uncertainty (v) and risk (u), the smooth ambiguity criterion emerges as a natural generalization of the SEU criterion as follows

$$V_{\mathrm{smooth}}(a_i) = \sum_{m_p \in M} \phi\Big(\sum_{s \in S} u\left(a_i(s)\right)\ m_p(s) \Big)\, \mu(m_p). \tag{4}$$

Under this framework, ambiguity aversion is characterized by a concave function ϕ, reflecting a more averse attitude towards the layer of model uncertainty than that of risk (i.e., v is more concave than u). Under the smooth ambiguity model, ambiguity aversion is thus characterized by an aversion to mean-preserving spreads (MPS) in expected utilities induced by different urn compositions and the prior distribution over them (Klibanoff et al., 2005).[10]

Under the smooth ambiguity family, the ambiguous situations presented in Table 1 can be ordered exclusively in terms of how much they are affected by ambiguity. For example, if the prior probability measure μ is uniform, one can show that an Ellsberg urn with n balls in it is more ambiguous than an urn with m balls as long as $n \leq m$ (see Berger, 2021, Proposition 3).[11] Thus, Ellsberg urns may be ranked as follows under the smooth ambiguity preference:

$$E1000 \succ E100 \succ E1 \iff R \succ Ej \quad \forall j \in \{1, 100, 1000\} \tag{5}$$

Maxmin models The second family of preference we analyze originates in the work of Gilboa and Schmeidler (1989). Their *multiple priors* approach relaxes the assumption of model uncertainty being quantified by a single probability measure μ and instead allows for the possibility of multiple priors belonging to a set C.[12] Under the α-maxmin expected utility criterion of Ghirardato et al. (2004), both the least favorable among all the classical subjective expected utilities determined by each prior μ in C and the most favorable one appear respectively with weights α and $1 - \alpha$. The multiple priors maxmin model of Gilboa and Schmeidler (1989) naturally emerges as a special case when $\alpha = 1$, while the classical SEU criterion is recovered when the set C contains only one element. When C consists of all possible prior probability measures, we recover the criterion due to Hurwicz (1951) when $\alpha \in (0, 1)$ and to Wald (1950) when $\alpha = 1$. In what follows, this is the version

[10]Note that Nau (2006) and Ergin and Gul (2009) characterize representations that, at least in special cases, can take the same representation as (4) and share the same interpretation as Klibanoff et al.'s (2005) version. Note that an alternative interpretation of the evaluation of an act under the smooth ambiguity model is the following: (1) in a first step, the DM computes, for each model m_p, a certainty equivalent $c(m_p)$ using her risk attitude captured by u, and (2) in a second step the DM evaluates the overall prospect by computing the expected utility over the different certainty equivalents using her prior μ and her attitude towards model uncertainty, captured by v. Model uncertainty aversion (i.e., concave v) is thus equivalent to an aversion to mean-preserving spreads in the space of certainty equivalents induced by each model.

[11]Note that when the identification of models is limited, there is no reason to assume a greater likelihood of a particular urn composition, e.g., that the composition 90-10 in $E100$ is more likely than the composition 60-40. A uniform prior measure ensures the same treatment for all compositions of the urn that are physically possible. It may be justified on the grounds of a general symmetry of information argument and is consistent with the principle of insufficient reason (Bernoulli, 1713; Laplace, 1814). As mentioned by Izhakian (2020), it is also consistent with the idea of the simplest non-informative prior in Bayesian probability (Bayes, 1763), and the principle of maximum entropy (Jaynes, 1957).

[12]This set of possible priors C incorporates both the attitude towards ambiguity and an information component: a smaller set C reflecting, for example, both better information and/or less ambiguity aversion.

we use. The utility of act i is

$$V_{\alpha-mxm}(a_i) = \alpha \min_{m_p} \Big(\sum_{s\in S} u\left(a_i(s)\right) m_p(s) \Big) + (1-\alpha) \max_{m_p} \Big(\sum_{s\in S} u\left(a_i(s)\right) m_p(s) \Big) \quad (6)$$

In this expression, α may be interpreted as an index of ambiguity attitude. For example, $\alpha = 0$ corresponds to a situation in which the DM is extremely optimistic and considers only the best possible composition of the urn, while $\alpha = 1$ corresponds to a DM being extremely pessimistic and considering only the worst possible composition. Under the maxmin family, it is easy to see that, while the Ellsberg situations are all more ambiguous than the risk R, we cannot order them according to the more ambiguous relation, as they all share the same worst ($p = 0$) and best ($p = 1$) possible models. Thus, irrespective of the degree of ambiguity aversion, we have, under the maxmin models:

$$E1 \sim E100 \sim E1000 \qquad \forall \alpha \in [0,1]\,. \quad (7)$$

Alternatively, a second approach for ordering situations by their degree of ambiguity has been recently proposed by Izhakian (2020). According to this appraoch, the degree of ambiguity of an act a_i may be quantified by its expected *volatility of probabilities*

$$\mho^2\left[a_i\right] \equiv \sum_{s\in S} \mathrm{E}_\mu\left[m_p^{a_i}(s)\right] \mathrm{Var}_\mu\left[m_p^{a_i}(s)\right], \quad (8)$$

where $m_p^{a_i}(s)$ is the probability of being in state s under model m, and $\mathrm{E}_\mu\,[.]$ and $\mathrm{Var}_\mu\,[.]$ are the expectation and variance operators, respectively, taken using the prior probability measure μ. The measure $\mho^2$ is claimed to be independent of attitudes towards risk and ambiguity, and has the advantage of being easily computable. The underlying decision-making model of this measure is the expected utility with uncertain probabilities (EUUP) model of Izhakian (2017), in which the preferences for ambiguity apply exclusively to the probabilities of events and are therefore outcome independent. Under this framework, a more ambiguous relation is defined as follows.

Definition: More Ambiguous (2) [Izhakian, 2020] Given two acts f, $g \in \mathrm{A}$ under which the expected probabilities of each consequence $c \in C$ are identical, f is a *more ambiguous* act than g if and only if

$$\mho^2[f] \geq \mho^2[g].$$

In words, an act g whose associated probabilities are on average less volatile than an act f is deemed less ambiguous. In a framework where ambiguity aversion takes the form of aversion to mean-preserving spreads in the space of probabilities, such an act is moreover preferred by any ambiguity averse individual.

In the context of our experiment, it should be clear that the symmetry condition ensures that the expected probabilities of the consequenes are the identical: $\mathrm{E}_\mu\left[m_p^{a_i}(s)\right] = \left[\sum_{m_p} m_p^{a_i}(s)\,\mu\left(m_p^{a_i}\right)\right] = 0.5$ for all $i \in \{R, E1, E100, E1000\}$ and all $s \in$ {red,black}.[13] The last column of Table 1 reports the expected volatility of probabilities for the situations we consider. As can be observed, $\mho^2$ is decreasing with the number of balls in Ellsberg ambiguous urns. The more ambiguous (2) relation thus predicts

$$E1000 \succ E100 \succ E1 \iff R \succ Ej \quad \forall j \in \{1, 100, 1000\} \tag{9}$$

which is similar to the more ambiguous (1) prediction under the smooth ambiguity model, summarized in expression (5).

3.4 The more complex relation

Some recent studies have supported the idea that ambiguity aversion (as, for example, captured by the ambiguity models presented above) may not be the only underlying factor behind the patterns typically observed in Ellsberg choices (Armantier and Treich, 2016; Kovářík et al., 2016; Aydogan et al., 2019). Instead these studies suggest that a separate notion of complexity aversion should be considered.

We are not aware of the existence of any formal economic theory capturing preferences for simple situations over complex ones under ambiguity.[14] Therefore, we rely on set theory to characterize the complexity of the situations. Remember that the DM's information about the likelihoods of the different states (and thus the outcome of the bet) is a priori modeled by the set $M = \{m_p \text{ such that } p \in I\}$, where p is the probability of winning (e.g., the probability that the ball drawn is red) and $I \subseteq [0, 1]$ is a set-theoretic modeling of information characterizing the chances to make a correct bet. Assuming that the DM has information about M, her acts needs to be measurable with respect to M without being allowed to condition the

[13] Note that, in line with Theorem 4 in Izhakian (2020), if the prior probability measure over the possible urn compositions is uniform, the condition, in our case, simply becomes: f is more ambiguous than g if and only if $\mathrm{Var}_\mu\left[m_p^f\right] \geq \mathrm{Var}_\mu\left[m_p^g\right]$, that is, the variance of the probabilities in f is higher than in g.

[14] Note that, under risk, an attempt is made by Puri (2018), who axiomatizes representations in which the DM assesses a lottery less favorably if it contains more outcomes.

choices on models that do not belong to M.

In analogy to what is done in the previous literature, the first definition we use relates complexity to the cardinality of M.[15]

Definition: More Complex (1) Given two acts f, $g \in \mathrm{A}$, f is a *more complex* act than g if $|M_f| \geq |M_g|$.

Under this intuitive definition, the degree of complexity of a situations depends exclusively on the *number of different potential models* the situations entails.[16] Using the more complex (1) relation, we can easily order the ambiguous situations presented in Table 1. Specifically, situation R is the less complex as the set M is singleton when the urn is not ambiguous. We directly observe that for ambiguous Ellsberg urns there exists a monotonic relation between the number of balls contained in the urn and the associated number of models, i.e., $|M| = N + 1$. Thus, if people are complexity averse, in the sense that they have a preference for simpler situations over more complex ones, we will observe:

$$R \succ E1 \succ E100 \succ E1000. \tag{10}$$

The second definition of *more complex* we present is based on the partial ordering of the sets M according to a "coarser than" ("finer than") relation.

Definition: More Complex (2) Assume that a binary relation "coarser than" is given as follows: M is coarser than M' (and M' is finer than M) if $I' \subseteq I$. Given two acts f, $g \in \mathrm{A}$, f is a *more complex* act than g if M_f is coarser than M_g.

In words, this definition means that more complex information regarding the structure of ambiguity may be naturally modeled by a larger set of potential models. In our case, the ordering between the sets is complete among the Ellsberg-type situation: $I_{E1} \subset I_{E100} \subset I_{E1000}$, but the more complex (2) relation is silent in comparing R and $E1$. Complexity averse individuals in the sense above will thus exhibit the following pattern of preference:

$$\begin{cases} E1 \succ E100 \succ E1000 \\ R \succ Ej \quad \forall j \in \{100, 1000\} \end{cases} \tag{11}$$

[15] Under risk, Sonsino et al. (2002); Moffatt et al. (2015) and Kovářík et al. (2016) define complexity as the number of different outcomes of a lottery.

[16] Note that this notion of complexity coincides with what Einhorn and Hogarth (1985, p. 435) referred to as the amount of ambiguity: "Moreover, the amount of ambiguity is an increasing function of the number of distributions that are not ruled out (or made implausible) by one's knowledge of the situation."

4 Results

4.1 Ambiguity attitudes with different Ellsberg urns

We first examine ambiguity attitudes by comparing situations of risk (i.e., when M is singleton) and ambiguity à la Ellsberg (when M contains several elements). Specifically, a subject is said to be ambiguity averse if she prefers the risky urn R over the ambiguous urn (either $E1$, $E100$ or $E1000$), ambiguity seeking if she prefers the ambiguous urn, and ambiguity neutral if she is indifferent between the two. Table 2 presents the results.

Table 2: Ambiguity Attitudes ($N = 84$)

Ambiguity Preferences	R vs. $E1$	R vs. $E100$	R vs. $E1000$
Ambiguity Aversion	32 (38.1%)	50 (59.5%)	42 (50%)
Ambiguity Neutrality	42 (50%)	26 (31%)	26 (31%)
Ambiguity Seeking	10 (11.9%)	8 (9.5%)	16 (19%)

We observe differences across the three ambiguous urns in the proportions of ambiguity aversion (Cochran's Q, p=0.009) and ambiguity neutrality (Cochran's Q, p=0.012). Specifically, ambiguity attitude measured with respect to $E1$ is different from that measured with respect to $E100$ and $E1000$. In particular, ambiguity aversion is the most common attitude under $E100$ and $E1000$, whereas there are more ambiguity neutral subjects than ambiguity averse ones under $E1$.[17] On the contrary, we do not observe any difference in ambiguity attitudes when measured with $E100$ and $E1000$.

We then test the association between Ellsberg ambiguity attitudes when using different urns. The contingency tables are presented in Table 3. As can be observed, we cannot reject the independence of ambiguity attitudes based on $E1$ from those based on $E100$ and $E1000$ (Fischer's exact test, p=0.178 and p=0.116, respectively). This suggests that the ambiguity attitudes measured with respect to a small-sized urn like $E1$ are distinct from those measured with respect to large-sized urns like $E100$ and $E1000$. Overall, the most common preference pattern is ambiguity neutrality under $E1$ together with ambiguity aversion under $E100$ or $E1000$. In contrast, we reject the independence hypothesis between ambiguity attitudes based on $E100$ and $E1000$ (Fischer's exact test, p=0.001), suggesting an

[17]The proportion of ambiguity neutrality is significantly lower under $E100$ and $E1000$ than under $E1$ (McNemar test, p=0.016 between $E1$ and $E100$, and p=0.018 between $E1$ and $E1000$), and the proportion of ambiguity aversion is higher under $E100$ than under $E1$ (McNemar test, p=0.009).

Table 3: Association between Ambiguity Attitudes

Part I: Ambiguity Attitudes using $E1$ and $E100$

Ambiguity preferences with $E1$	**Ambiguity preferences with $E100$**			**Total**
	Aversion	Neutrality	Seeking	
Aversion	17	12	3	**32**
Neutrality	28	12	2	**42**
Seeking	5	2	3	**10**
Total	**50**	**26**	**8**	**84**
Independence test:	Fischer's exact test (2-sided): $p = 0.178$			

Part II: Ambiguity Attitudes using $E1$ and $E1000$

Ambiguity preferences with $E1$	**Ambiguity preferences with $E1000$**			**Total**
	Aversion	Neutrality	Seeking	
Aversion	11	11	10	**32**
Neutrality	26	11	5	**42**
Seeking	5	4	1	**10**
Total	**42**	**26**	**16**	**84**
Independence test:	Fischer's exact test (2-sided): $p = 0.116$			

Part III: Ambiguity Attitudes using $E100$ and $E1000$

Ambiguity preferences with $E100$	**Ambiguity preferences with $E1000$**			**Total**
	Aversion	Neutrality	Seeking	
Aversion	34	10	6	**50**
Neutrality	6	13	7	**26**
Seeking	2	3	3	**8**
Total	**42**	**26**	**16**	**84**
Independence test:	Fischer's exact test (2-sided): $p = 0.001$			

association between ambiguity attitudes measured with large-sized Ellsberg urns. Overall, we summarize this first set of results as follows.

Result 1: *Ambiguity attitudes measured with the small-sized urn $E1$ are different and independent from those measured with the large-sized urns $E100$ and $E1000$. In contrast, ambiguity attitudes measured with large-sized urns $E100$ and $E1000$ are comparable and associated with each other.*

4.2 Direct preferences over Ellsberg urns

In this section, we focus on comparing directly ambiguous Ellsberg urns of different size. Table 4 reports the results of the three pairwise comparisons between $E1$, $E100$, and $E1000$. First, we observe that, for every pairwise comparison, the

Table 4: Size Preferences under Ambiguity

	$E1$ vs. $E100$	$E1$ vs. $E1000$	$E100$ vs. $E1000$	Classification (majority rule)
Prefer larger urn	29 (34.5%)	39 (46.4%)	30 (35.7%)	38 (46.3%)
Indifferent	19 (22.6%)	26 (31%)	38 (45.2%)	23 (28%)
Prefer smaller urn	36 (42.9%)	19 (22.6%)	16 (19%)	21 (25.6%)
Total	84	84	84	82

majority of the subjects is *not* indifferent to the size of the urn on which to bet. Second, focusing on strict preferences, we find that a preference for larger urns is more common than for smaller urns in the comparisons $E1$ vs. $E1000$ and $E100$ vs. $E1000$ (one-sample test of proportion, p=0.018 and p=0.039, respectively) whereas the preference for the small urn in the comparison $E1$ vs. $E100$ is not significant (one-sample test of proportion, p=0.385).

Next, we provide an individual level analysis by classifying subjects according to their direct preferences over Ellsberg urns. Using a majority rule, we classify subjects as exhibiting a preference for large-sized (small-sized) urns if they prefer the larger (smaller) urn in at least two pairwise comparisons (out of three possible). Similary, subjects are classified as indifferent if she exhibits indifference in at least two out three pairwise comparisons. Two subjects whose preferences do not exhibit any dominant pattern remain unclassified. The last column of Table 4 presents the result of the classification. The proportion of subjects exhibiting a strict preference for the size of ambiguous Ellsberg urns (either large- or small-sized urns) is 72%. Among those subjects, we observe that the majority prefers larger urns over smaller ones (one-sample test of proportion, p=0.027), which is consistent with the results obtained previously in the literature (Filiz-Ozbay et al., 2021). We summarize our second set of findings as follows.

Result 2: *The majority of the subjects exhibits preferences over Ellsberg urns with different sizes, and larger urns tend to be preferred to smaller urns.*

4.3 Explaining preferences over Ellsberg urns

We now turn to exploring our data in the light of the predictions presented in Section 3. Following the "more ambiguous" and "more complex" definitions, different patterns can be distinguished.

1. An observed preference for large-sized Ellsberg urns over small-sized ones can be related either to *(i) smooth ambiguity aversion* (in which case the subject exhibits ambiguity aversion), or to *(ii) complexity seeking* (in which case the subject also exhibits ambiguity seeking).

2. An observed preference for small-sized Ellsberg urns over large-sized ones can be related either to *(i) complexity aversion* (in which case the subject exhibits ambiguity aversion), or to *(ii) smooth ambiguity seeking* (in which case the subject exhibits ambiguity seeking).

3. Indifference towards the size of Ellsberg urns can be related either to *(i) SEU* (in which case the subject is also ambiguity neutral), or to *(ii) maxmin* preferences.

In what follows, we classify our subjects based on these theoretical patterns. To do this, we look at the interaction between the preferences for large- or small-sized Ellsberg urns and the ambiguity preferences. For the preferences over Ellsberg urns, we use the individual level classification reported in the previous section, which is based on the majority rule. For ambiguity preferences, we also use a majority rule that classifies a subject as ambiguity averse (resp. seeking, neutral) if she exhibits ambiguity aversion (seeking, neutrality) in at least two out of the three comparisons between R and the Ellsberg urns $E1$, $E100$ and $E1000$.[18] We do not include the seven subjects whose preferences cannot be determined by the majority rules in the analysis.

[18]Note that our definition of more complex (2) relies solely on ambiguity preferences comparing R with $E100$ and $E1000$. While we use here the majority rule relying on all three Ellsberg urns based on definition of more complex (1), we further report the cases where the subject would remain unclassified based on more complex (2), specifically when the subject exhibits at the same time ambiguity averse and seeking attitudes with respect to $E100$ and $E1000$ (e.g. $R \succ E100$ & $E1000 \succ R$).

The results of this classification are reported in Table 5. In total, 63 out of 77 subjects (82%) are classified as behaving in accordance with one of the aforementioned theoretical patterns (see highlighted cells in Table 5). First, we observe that 42.9% (=27/63) of the classified subjects behave in accordance with either the smooth or the maxmin ambiguity aversion hypothesis. Alternatively, 28.6% (=18/63) of the classified subjects exhibit complexity aversion.

Table 5: Classification of Subjects Based on Ambiguity and Size Preferences

Preferences for the size of Ellsberg urns	**Ambiguity preferences** Averse	Neutral	Seeking	**Total**
Prefer larger urns	16 (20.8%) [Smooth AA]	11 (14.3%)	7 (9.1%) [Complexity Seeker]	**34** (44.15%)
Indifference	11 (14.3%) [Maxmin AA]	11 (14.3%) [SEU]	0 (0%) [Maxmin AS]	**22** (28.6%)
Prefer smaller urn	18 (23.4%) [Complexity Averse]	3 (3.9%)	0 (0%) [Smooth AS]	**21** (2.7%)
Total	**45** (58.4%)	**25** (32.5%)	**7** (9.1%)	**77**

Interestingly, we also observe that 11.1% (=7/63) of the subjects are classified as exhibiting complexity seeking,[19] whereas we do not find any preferences consistent with the smooth or maxmin ambiguity seeking hypotheses. Finally, the proportion of subjects following the SEU hypothesis amounts to 17.5% (=11/63).

Result 3: *Attitudes towards both ambiguity and complexity play a role in the heterogeneity of preferences over different Ellsberg urns.*

4.4 Robustness analysis and further results

It may be argued that subjects could have been affected by other features. Among others, the number of balls in the different uncertain situations may drive some of our results. Ratio bias has been defined in Denes-Raj et al. (1995) as the

[19]Among 25 subjects who are classified as complexity averse or seeking, 22 of them were also consistent with definition of more complex (2), i.e., not exhibiting any conflicting ambiguity attitudes with respect to $E100$ and $E1000$.

fact that subjects tend to consider odds more favorably when they are represented by a ratio of larger (10 in 200) rather than smaller (1 in 20) absolute numbers.[20] Among the reasons evoked to explain such a bias, denominator neglect has been considered as crucial. Subjects tend to focus on numerator and consider only the frequency of "success" and neglect the total frequency. In this section, we test the potential confounding effect of the number of balls in the urns, which alters the denominators of possible urn compositions. To check this effect, we consider the situation $P100$. Recall that $P100$ has the same set of models as $E1$, but contains different number of balls (i.e., 100 balls in $P100$ vs 1 ball in $E1$). On the contrary, $P100$ has the same number of balls as $E100$, but not the same set of potential models. In the absence of ratio bias, subjects should be indifferent between E1 and P100, while they could exhibit preferences between E100 and P100.

Table 6 reports the association between ambiguity attitudes comparing R with $P100$ and $E1$, respectively. As can be observed, the majority of the observations

Table 6: Association between Ambiguity Attitudes with $E1$ and $P100$

Ambiguity preferences with $E1$	**Ambiguity preferences with $P100$**			**Total**
	Aversion	Neutrality	Seeking	
Aversion	26	3	3	**32**
Neutrality	10	28	4	**42**
Seeking	4	3	3	**10**
Total	**40**	**34**	**10**	**84**
Independence test:	Fischer's exact test (2-sided): $p < 0.001$			

(57 out of 84, 67.9%) is on the diagonal and the association between ambiguity attitudes when measured with these two sources is highly significant. Although the proportion of neutrality is somewhat higher and aversion lower when $E1$ is considered rather than $P100$, the difference is not significant at 5% level (McNemar test, p=0.074).

We then test for direct preferences between $P100$ and $E1$ and between $P100$ and $E100$. Table 7 reports the results. We observe that a large majority of subjects (70.2%) shows indifference between $P100$ and $E1$. On the contrary, a majority of subjects has a strict preference when $P100$ is compared to $E100$. These results suggest that the effect solely due to the number of balls contained in the urn, if any, is limited.

To further examine the robustness of our results testing the impacts of the "more ambiguous" and "more complex" relations, while at the same time controlling

[20]First evidence of ratio bias has been found in Kirkpatrick and Epstein (1992). Ratio bias has been document in different sciences such as medicine (e.g., Yamagishi, 1997) or political sciences (e.g., Pedersen, 2017).

Table 7: PREFERENCES FOR $P100$ AGAINST $E1$ AND $E100$ ($N = 84$)

	$P100$ vs. $E1$	$P100$ vs. $E100$
Prefer $P100$	10 (11.9%)	34 (40.5%)
Indifferent	59 (70.2%)	17 (20.2%)
Prefer Ellsberg urn	15 (17.9%)	33 (39.3%)

for the differences in number of balls in the urns, we run a conditional logistic regression analysis. The analysis is performed by pooling the data from the nine binary comparisons included in the experiment. The independent variables are the "degree of complexity", measured by the cardinality of the set of models M in accordance with the more complex (1) relation, and the "degree of ambiguity", measured by the expected volatility of probabilities in accordance with the more ambiguous (2) relation. We control for the denominator effect using using the variable "total number of balls" in the urns. To account for the potential non-linearities in the impact due to the number of models and balls on choices, we use the log of these variables.[21] The results of the regressions are shown in Table 8.

Table 8: CONDITIONAL LOGISTIC REGRESSION

	(1)	(2)
Degree of complexity	-0.114**	-0.147***
	(0.037)	(0.036)
Degree of ambiguity	-4.574***	-3.881***
	(0.806)	(0.942)
Denominator effect (number of balls)		0.0708
		(0.031)
Observations	938	938

Notes: The analysis consist of 18 data points (9 pairs of bets) for each of the 84 subjects. The indifferent observations are automatically dropped from the analysis as the contribution of these observations to log-likelihood is zero. Robust standard errors, cluster-corrected at individual level in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression (1), which does not include control, indicates that more complex sources (i.e. sources with a higher number of potential models) and more ambiguous sources (i.e., sources with a higher spread in the potential models) are less preferred. This result confirms the hypothesis that both complexity and ambiguity aversion are at

[21] The underlying assumption is that going from 1 model (resp. ball) to 2 models (balls) possibly has a larger effect than going from 100 models (balls) to 101 models (balls). Note that, by choosing a log transformation, we assume linearity over percentage changes. These assumptions are supported in our estimations by both AIC and BIC scores.

play comparing different urns. Regression (2) includes the number of balls in the analysis. We find that controlling for the number of balls does not alter the impact and significance of the regression coefficients. The coefficient of the number of balls is positive, indicating that Ellsberg urns with higher number of balls tend to be more preferred, although this effect is only marginally significant at 10% level (p=0.054). Based on this analysis, we conclude that the main driver of preferences captured in our experiment is due to the differences in the set of potential models rather than to the differences in the number of balls in the urns. These results moreover are robust to alternative prior measure μ (for futher details, see Appendix A).

5 Related experimental literature

Our article is closely related to a recent study by Filiz-Ozbay et al. (2021). While both designs share certain common features (e.g., the comparison between risky urns and Ellsbergs' ambiguous urns of different sizes and the method used to elicit preferences), the topic and therefore the analysis of the two studies differ. In our paper, we show the importance of two features of urns (namely, complexity and spread) on decision making while their study investigates preferences for the size of the ambiguous urn in relation to modern ambiguity models and analyzes the role of ratio bias. In this section, we reexamine the experiment of Filiz-Ozbay et al. (2021) from the perspective of our analysis with degrees of ambiguity and complexity.[22]

Filiz-Ozbay et al. (2021) use binary choices to elicit preferences between different urns. They consider three risky urns and three ambiguous urns with 2 balls, 10 balls and 1000 balls (using our notation, R2, R10, R1000 and E2, E10 and E1000). They elicit ambiguity preferences using the comparison between risky urns and ambiguous urns of the same size (i.e., R2vsE2, R10vsE10 and R1000vsE1000) and elicit direct size preferences by comparing E2vsE10, and E10vsE1000. To connect the observed preferences to our theoretical predictions, we replicate our classification in Table 6 by using their data. Following the same procedures, we use a majority rule to classify their subjects according to their ambiguity preferences and their preferences towards the size of the urns. The results are in Table 9. Their data indicate somewhat different patterns than ours. While we find heterogeneity in preferences in our dataset indicating both complexity and ambiguity attitudes, in their data, we find a large majority (60.8%) of smooth ambiguity averse agents but a very small share of patterns suggeting complexity attitudes (5.4%). Altogether, the results based on this dataset suggest that a majority of subjects seems to be spread averse and indifferent to complexity.

[22]We kindly thank the authors of the study for having shared their data with us.

Table 9: Classification of Subjects Based on Ambiguity and Size Preferences in Filiz-Ozbay et al. (2021)

Preferences for the size of Ellsberg urns	**Ambiguity preferences** Averse	Neutral	Seeking	**Total**
Prefer larger urns	45 (60.8%) [Smooth AA]	3 (4.05%)	2 (2.7%) [Complexity Seeker]	**50** (67.6%)
Indifference	11 (14.9%) [Maxmin AA]	10 (13.5%) [SEU]	1 (1.35%) [Maxmin AS]	**22** (29.7%)
Prefer smaller urn	2 (2.7%) [Complexity Averse]	0 (0%)	0 (0%) [Smooth AS]	**2** (2.7%)
Total	**58** (78.4%)	**17.6** (4.05%)	**3** (3.9%)	**74**

The differences between the two results might possibly be due to subtle differences in stimuli and design. While their data contains only 5 binary choices (3 measures of ambiguity preferences and 2 measures of direct size preferences),we in our experiment had 9 binary choices (4 measures of ambiguity preferences, 3 measures of direct size preferences and 2 measures of framing). We think that the variety of scenarios used in our experiment could allow for more heterogeneity in preferences. We also believe that the urns under consideration in the two experiments may also explain the difference in results. In particular, we compare one small urn with two large urns (E1 and E100 + E1000), whereas they compare either two small urns or one small urn and a large urn (E2 + E10 vs E1000). While our measures (E1 vs E100 and E1 vs E1000) offer a more serious tradeoff between complexity and spread, their measures (E2 vs E10 and E10 vs E1000) may have made this tradeoff less salient.

Moreover, E1 is a very specific urn as it is at the same time the ambiguous urn with the highest degree of spread and the minimal degree of complexity. In our experiment, it is the only urn for which ambiguity aversion is less frequent than ambiguity neutrality. Notably, in Filiz-Ozbay et al. (2021), in all the three ambiguous urns, ambiguity aversion is always more frequent than ambiguity neutrality. Furthermore, in our results 1 and 2, we show that the pattern observed with E1 differs from the pattern observed with E100 and E1000. Yet, the pattern with E2 does not differ from the pattern observed with E10 and with E1000 in Filiz-Ozbay et al. (2021). Altogether, this suggest that a E1 effect may exist and explain the difference in results.

6 Concluding remarks

In this paper, we offer an original approach to study choices over Ellsbergs urns based on (i) ambiguity preferences and (ii) complexity preferences. For ambiguity, we use two existing definitions of "more ambiguous " relation in the literature (one based on Jewitt and Mukerji (2017) and another based on Izhakian (2020). Furthermore, we propose two new definitions of complexity under model uncertainty. The definitions provide us with ready-to-use measures, which are shown to be useful while examining preferences under model uncertainty. Building on these definitions, we are able to derive theoretical predictions about size preferences, and experimentally test these predictions using ambiguous-Ellsberg-urns made of 1, 100 and 1000 balls. Our results show that subjects do not exhibit the same ambiguity atittudes when preferences are measured with different sized ambiguous urns, and they exhibit size preferences with a tendency to prefer larger urns to smaller ones. Importantly, we observe that there exists strong heterogeneity of preferences over Ellsberg urns and that both complexity and ambiguity attitudes are important features for explaining those preferences. The tractable measures of ambiguity and complexity used in this study in the Ellberg situations are expected to be useful in contexts more general than the Ellsberg paradigm and help shedding more light on the effect of those factors on decisions under uncertainty in future studies.

Appendix

A Alternative prior measure

While assuming a uniform prior probability measure μ seems the most natural way to proceed when the information available is limited to identify the correct urn composition, we here investigate the robustness of our findings to alternative prior measures. In particular, we study two new cases where the prior measure follows either a binomial distribution or a hypergeometric distribution. Regarding a prior measure that follows a binomialdistribution is equivalent to treating each ball in the urn as being equally likely to be red or black. Hypergeometric prior can be seen as picking all balls randomly from 2 jars made of N balls.

Column 2 of Table A.1 reports the expected volatility of probabilities $\mho^2$ (of drawing either a red or a black ball) under a binomial prior μ (as computed by equation (8)) while column 3 reports $\mho^2$ under a hypergeometric prior μ . Similar to the uniform prior, $\mho^2$ is decreasing with the number of balls in Ellsberg ambiguous urns, thus leaving our predictions unchanged.

Table A.1: Degree of ambiguity under binomial prior

Uncertain situation	Volatility of probabilities ($\mho^2$) binomial prior	Volatility of probabilities ($\mho^2$) hypergeometric prior
R	0.0000	0.0000
$E1$	0.2500	0.2500
$E100$	0.0025	0.00125
$E1000$	0.0025	0.0001
$P100$	0.2500	0.2500

We then replicate the analysis of Section 4.4. The results of the conditional logistic regression are presented in Table A.2.

In line with what was previously reported using uniform priors, we observe that more complex and more ambiguous sources are less preferred. We also find that controlling for the number of balls does not alter the impact and significance of the regression coefficients. The coefficient associated with the denominator effect is, this time, significant.

Table A.2: CONDITIONAL LOGISTIC REGRESSION: BINOMIAL PRIOR

	Binomial		**Hypergeometric**	
	(1)	(2)	(3)	(4)
Degree of complexity	-0.165***	-0.199***	-0.165***	-0.200***
	(0.039)	(0.038)	(0.039)	(0.038)
Degree of ambiguity	-3.888***	-3.098***	-3.866***	-3.072***
	(0.721)	(0.815)	(0.719)	(0.811)
Denominator effect (number of balls)		0.095**		0.096**
		(0.035)		(0.035)
Observations	938	938	938	938

Notes: The analysis consist of 18 data points (9 pairs of bets) for each of the 84 subjects. The indifferent observations are automatically dropped from the analysis as the contribution of these observations to log-likelihood is zero. Robust standard errors, cluster-corrected at individual level in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

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Online Appendix

S1 Experimental details: Instructions

Welcome page and examples

Welcome to this experiment!

You are participating in a study on decision making under uncertainty. The experiment will take approximately 60 minutes.

You will be confronted with different uncertain scenarios. Each uncertain scenario involves urns from which a ball is drawn randomly. In each scenario, you are asked to make choices between two options.

In each uncertain scenario, you will be the one who chooses the color of the ball on which to bet to potentially win a gain in €.

>>

There are 14 *uncertain scenarios* in total. In every uncertain scenario, the urns contain only red and black balls. There is **no** other color.

The urns were constructed before the experiment by one of the collaborators, who is not present in the room now.

The urns associated with different uncertain scenarios are (visibly) placed on the table in front of the room. The experimenters also have no information about their compositions except the descriptions provided to you in the instructions. You will be invited to check them at the end of the experiment.

Next, you will see some examples of uncertain scenarios.

>>

Example 1

There are **100** balls in an urn. Each of the 100 balls in the urn is either black or red. The proportion of red and black balls is **unknown**.

A ball will be randomly drawn from the urn and its color will be observed.

Suppose you have chosen black as the color associated with winning €15.

The following list contains 16 choice questions. On each line, you are asked to make a decision between **Option 1** (on the left) and **Option 2** (on the right). **Option 1** gives you €15 if the ball drawn is black and €0 otherwise. **Option 2** gives you a sure amount of money ranging between €15 and €0.

Notice that **Option 2** gives you sure amounts that are decreasing as you move down the table whereas **Option 1** is the same throughout. For example, **Option 2** gives you a sure €15 on line 1, and €0 for sure on line 16.

You indicate your decision on each line by marking the circle next to the option.

Option 1			Option 2
Have €15 if the ball drawn is black	○	○	Have €15 for sure
Have €15 if the ball drawn is black	○	○	Have €14 for sure
Have €15 if the ball drawn is black	○	○	Have €13 for sure
Have €15 if the ball drawn is black	○	○	Have €12 for sure
Have €15 if the ball drawn is black	○	○	Have €11 for sure
Have €15 if the ball drawn is black	○	○	Have €10 for sure
Have €15 if the ball drawn is black	○	○	Have €9 for sure
Have €15 if the ball drawn is black	○	○	Have €8 for sure
Have €15 if the ball drawn is black	○	○	Have €7 for sure
Have €15 if the ball drawn is black	○	○	Have €6 for sure
Have €15 if the ball drawn is black	○	○	Have €5 for sure
Have €15 if the ball drawn is black	○	○	Have €4 for sure
Have €15 if the ball drawn is black	○	○	Have €3 for sure
Have €15 if the ball drawn is black	○	○	Have €2 for sure
Have €15 if the ball drawn is black	○	○	Have €1 for sure
Have €15 if the ball drawn is black	○	○	Have €0 for sure

>>

Below, you are asked some comprehension questions testing your understanding of Example 1. Please indicate your answers.

Example 1

Each of the 100 balls in the urn is either black or red. The proportion of red and black balls is unknown.

Q. Please indicate whether the following sentence is true or false for Example 1.

The number of **red balls** in the urn may be any number between 0 and 100.

○ True ○ False

It is possible that the urn is composed of only **red balls**.

○ True ○ False

It is possible that the urn is composed of only **black balls**.

○ True ○ False

It is possible that the urn is composed of an equal proportion of **red and black balls**.

○ True ○ False

The urn might contain a ball of a color other than red and black.

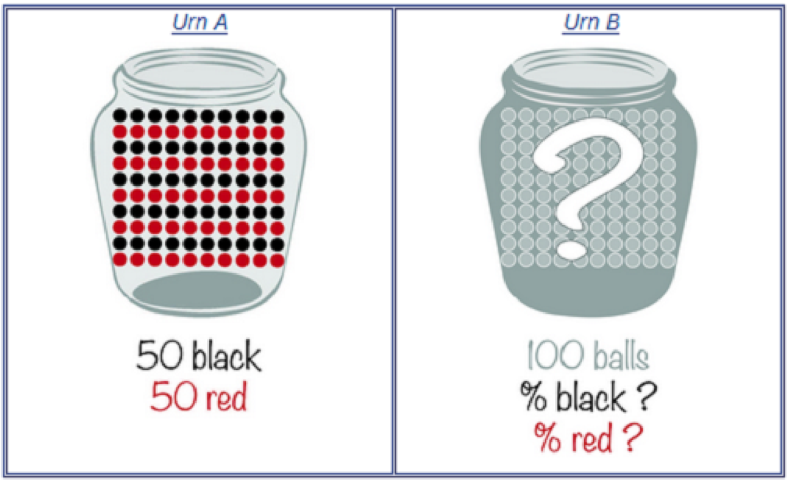
○ True ○ False

>>

Example 2

There are two urns with balls that can either be red or black.

- Urn A is composed of **50 Red** and **50 Black** balls.
- Urn B contains **100** balls. The proportion of red and black balls is **unknown**.



A ball will be randomly drawn from each of the two urns and its color will be observed.

You are asked to place a bet on the color of the ball drawn from one of the two urns. If your bet is correct, you will receive an amount of money in €. If it is not correct, nothing happens.

Suppose you have chosen black as the color associated with winning a gain in €.

The following line contains 1 choice question. You are asked to make a decision between 2 options.
Option 1 (on the left) gives you €15 if the ball drawn from **Urn A** is black and €0 otherwise.
Option 2 (on the right) gives you €15 if the ball drawn from **Urn B** is black and €0 otherwise.

You indicate your decision by marking the circle below the option.

On which urn do you want to bet?

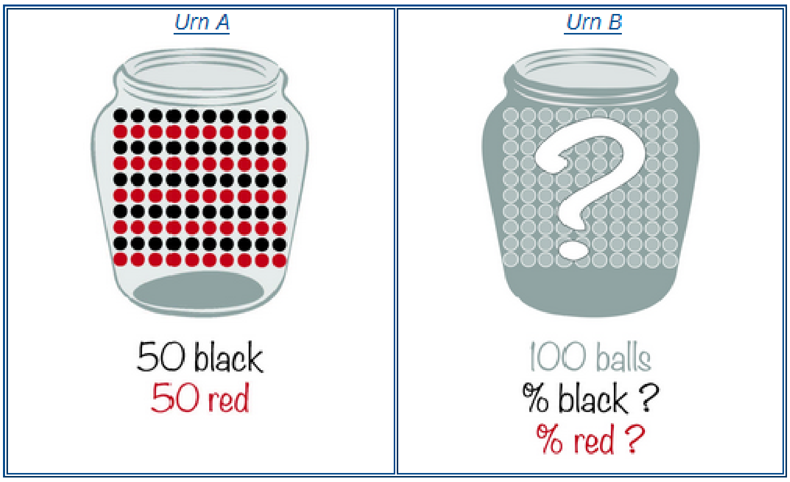
Option 1	Option 2
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Below, you are asked some comprehension questions testing your understanding of Example 2. Please indicate your answers.

Example 2

- Urn A is composed of **50 Red** and **50 Black** balls.
- Urn B contains **100** balls. Each of the 100 balls is either **black** or **red**. The proportion of red and black balls is **unknown**.



Q. Please indicate whether the following sentence is true or false for Example 2.

It is possible that Urn A and Urn B have exactly the same composition with **50% black** and **50% red balls.**

○ True ○ False

It is possible that there are more **red balls** in Urn B than in Urn A.

○ True ○ False

It is possible that there are more **black balls** in Urn B than in Urn A.

○ True ○ False

It is possible that there are more **black balls** and more **red balls** in Urn B than in Urn A.

○ True ○ False

Urn B might contain a ball of a color other than red and black.

○ True ○ False

>>

Before starting the incentivized part of the experiment, we clarify how subjects were going to be paid.

Your payment

Each of you will receive €5 for your participation.
In addition, you will also be paid based on one of your decisions in the experiment.

Which decision will matter for your payment?

During this experiment, you will make decisions in 14 different uncertain scenarios (of type I and II).
Type I sceanrios: in 5 scenarios, you will face a list of 16 choice questions (as in the tables we showed you in Example 1). Thus, there are 80 choice questions in these scenarios in total.
Type II sceanrios: in 9 scenarios, you will face 2 choice questions (as the one we showed you in Example 2). Thus, there are 18 choice questions in these scenarios in total.
In total, there are thus 98 choice questions.

One of the 98 choice questions will be picked in the following manner: We have 98 sealed envelopes each containing one of the 98 choices you will face in the 14 uncertain scenarios. We will randomly pick one envelope. The uncertain scenario and the choice question there contained, as well as your corresponding recorded decision in that choice question, will determine your payment.

The content of the envelope picked will be revealed to you only at the end of the experiment. Hence, each of the 98 choice questions has an equal chance of being picked. Therefore, in every choice question it is in your best interest to make your decisions whilst keeping in mind that it could be the one determining your payment at the end.

How will the payments be undertaken?

At the end of the experiment:
1. The picked envelope containing the uncertain scenario and the choice questions is opened.
2. The urn(s) in the relevant uncertain scenario is/are selected. A ball is drawn from the urn(s).
3. Your recorded decision in the relevant choice question is observed. Your payment is given based on the option you chose.

>>

Payment example 1

For example, suppose the envelope contains the following scenario and choice question:

Uncertain Scenario X, Choice Question 1

There are **100** balls in an urn. Each of the 100 balls in the urn is either black or red. The proportion of red and black balls is **unknown**.

A ball will be randomly drawn from the urn and its color will be observed.

Suppose you have chosen black as the color associated with winning €15.

Option 1			Option 2
Have €15 if the ball drawn is black	⦿	○	Have €15 for sure

Furthermore, suppose you chose Option 1. Then, you are paid €15 if the color of the ball drawn from the urn is black and 0€ otherwise.

In the case you chose Option 2, you would have received €15 for sure.

>>

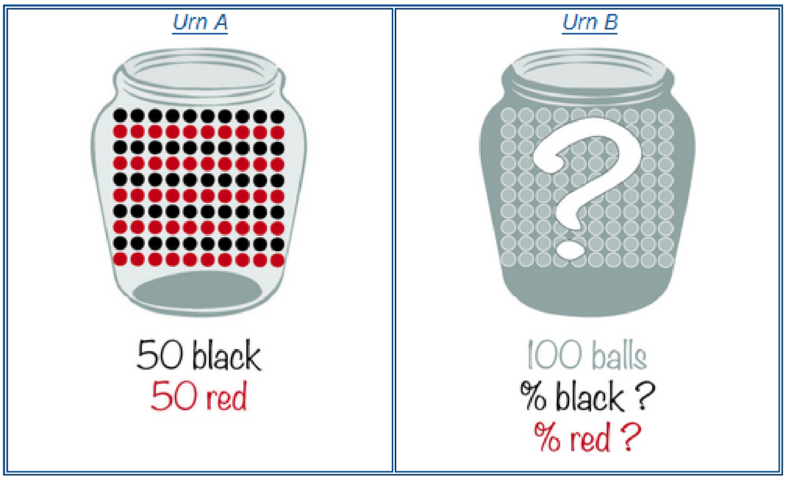
Payment example 2

For example, suppose the envelope contains the following scenario and choice question:

Uncertain Scenario Y, Choice Question 1

There are two urns with balls that can either be red or black.

- Urn A is composed of **50 Red** and **50 Black** balls.
- Urn B contains **100** balls. The proportion of red and black balls is **unknown**.



A ball will be randomly drawn from one of the two urns and its color will be observed.
Suppose you have chosen black as the color associated with winning a gain in €.

Option 1	Option 2
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct

Furthermore, suppose you chose Option 2. Then, you are paid €15 if the color of the ball drawn from Urn B is black and 0€ otherwise.

In the case you chose Option 1, you would have received €15 if the color of the ball drawn from Urn A were black and 0€ otherwise.

>>

Every subject made decisions in 14 different scenarios in total (9 scenarios taking the form of random lottery pairs and 5 scenarios taking the form of certainty equivalents). The order in which the type of scenarios appears is randomized.

Random lottery pairs

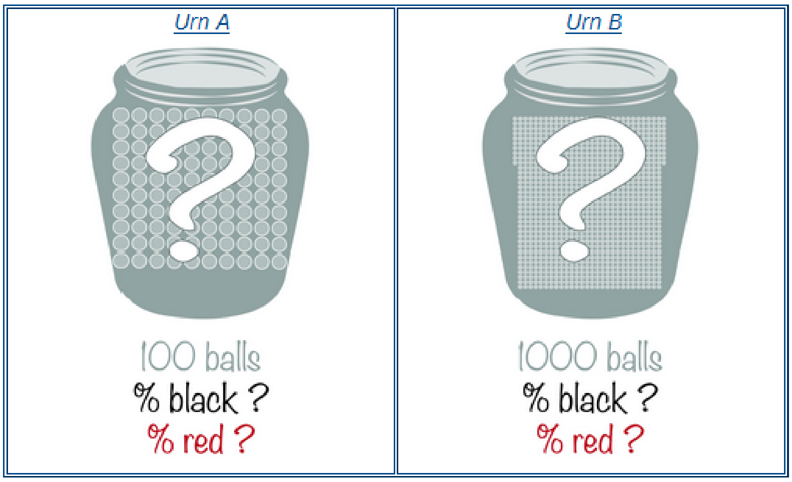
Each of the 9 scenarios presents two choice questions, presented in a sequence. The order in which the scenario appear is randomized.

Question 1

Uncertain scenario θ

There are two urns with balls that can either be red or black.

- *Urn A contains **100** balls. The proportion of red and black balls is **unknown**.*
- *Urn B contains **1000** balls. The proportion of red and black balls is **unknown**.*



A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

- ○ I would like to receive money if the ball drawn is black
- ○ I would like to receive money if the ball drawn is red

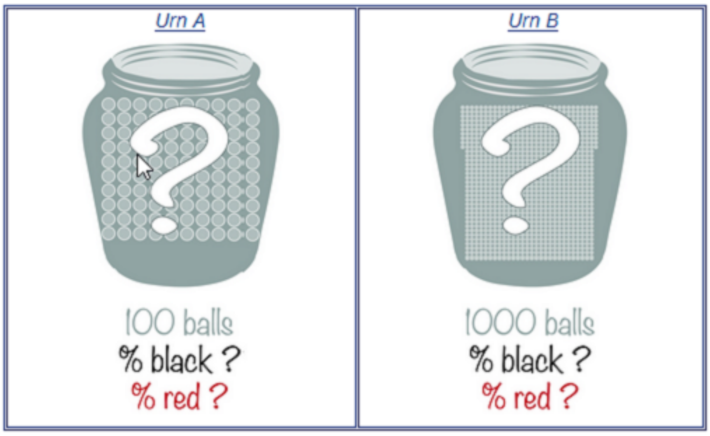
You will receive €15 if your bet is correct.
On which urn do you want to bet?

I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Question 2 If Urn A is chosen first:

- Urn A contains **100** balls. Each of the 100 balls is either **black** or **red**. The proportion of red and black balls is **unknown**.
- Urn B contains **1000** balls. Each of the 1000 balls is either **black** or **red**. The proportion of red and black balls is **unknown**.



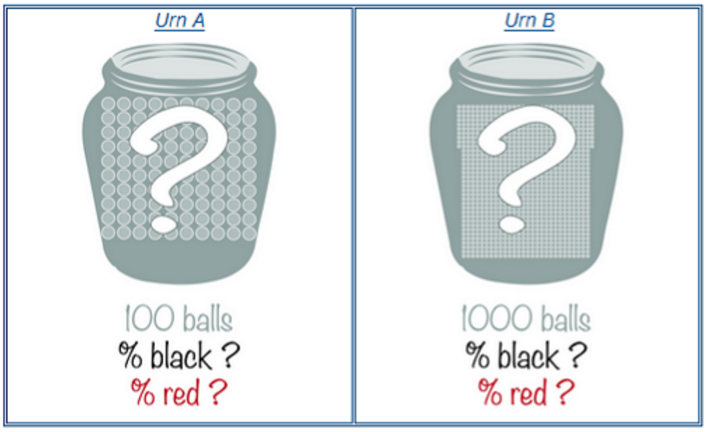
You chose to bet on Urn A.
Now, we ask you to reconsider your choice if a correct bet on Urn B gives a slightly higher gain.

On which urn do you want to bet:

I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15.10 if my bet is correct
○	○

Question 2 If Urn B is chosen first:

- Urn A contains **100** balls. Each of the 100 balls is either **black** or **red**. The proportion of red and black balls is **unknown**.
- Urn B contains **1000** balls. Each of the 1000 balls is either **black** or **red**. The proportion of red and black balls is **unknown**.



You chose to bet on Urn B.
Now, we ask you to reconsider your choice if a correct bet on Urn A gives a slightly higher gain.

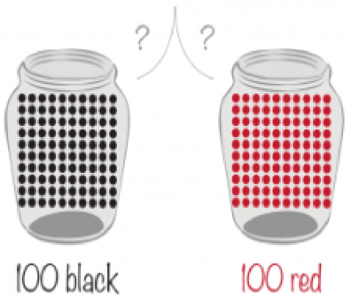
On which urn do you want to bet:

I would like to bet on Urn A, and receive €15.10 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

Uncertain scenario λ

There are two urns with balls that can either be red or black.

- Urn A contains **100** balls. The proportion of red and black balls is **unknown**.
- Urn B contains **100** balls. The balls are either **all black** or **all red.** The exact composition is **unknown**.

Urn A	Urn B
	

A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

- ○ I would like to receive money if the ball drawn is black
- ○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

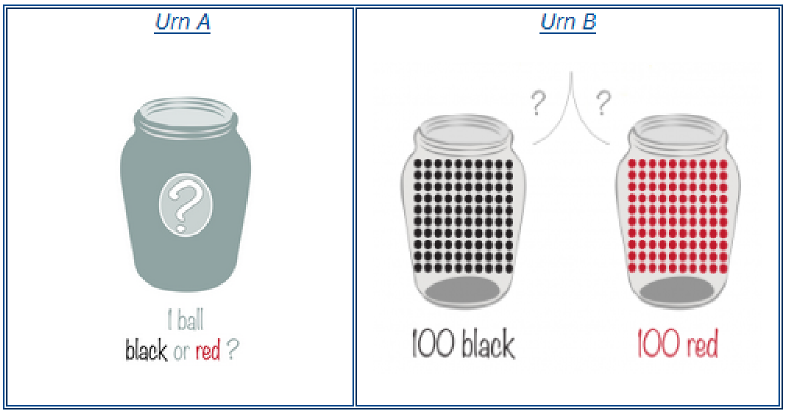
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Uncertain scenario ζ

There are two urns with balls that can either be red or black.

- Urn A contains **1** ball. The color of the ball is **unknown**.
- Urn B contains **100** balls. The balls are either **all black** or **all red.** The exact composition is **unknown**.



A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

○ I would like to receive money if the ball drawn is black

○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

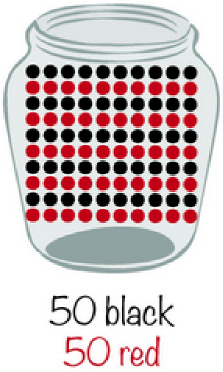
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Uncertain scenario α

There are two urns with balls that can either be red or black.

- *Urn A is composed of **50 Red** and **50 Black** balls.*
- *Urn B contains **1** ball. The color of the ball is **unknown**.*

Urn A	*Urn B*
	

A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

I would like to receive money if the ball drawn is black

I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

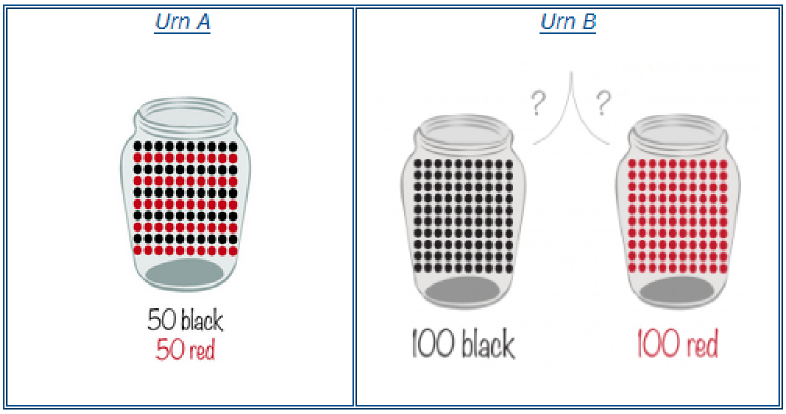
I would like to bet on Urn A, and receive €15 if my bet is correct | I would like to bet on Urn B, and receive €15 if my bet is correct

>>

Uncertain scenario 5

There are two urns with balls that can either be red or black.

- Urn A is composed of **50 Black** and **50 Red** balls.
- Urn B contains **100** balls. The balls are either **all Black** or **all Red.** The exact composition is **unknown**.



A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

○ I would like to receive money if the ball drawn is black

○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

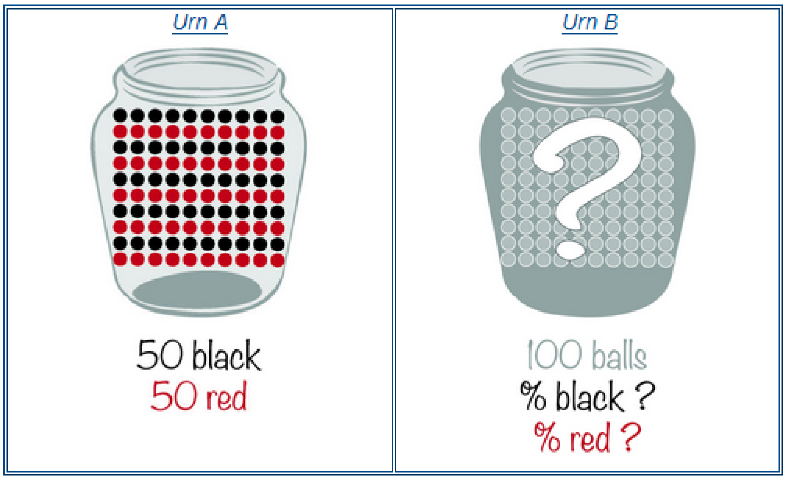
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Uncertain scenario β

There are two urns with balls that can either be red or black.

- *Urn A is composed of **50 Red** and **50 Black** balls.*
- *Urn B contains **100** balls. The proportion of red and black balls is **unknown**.*



A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

- I would like to receive money if the ball drawn is black
- I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

- I would like to bet on Urn A, and receive €15 if my bet is correct
- I would like to bet on Urn B, and receive €15 if my bet is correct

>>

Uncertain scenario η

There are two urns with balls that can either be red or black.

- *Urn A contains **1** ball. The color of the ball is **unknown**.*
- *Urn B contains **100** balls. The proportion of red and black balls is **unknown**.*

Urn A	*Urn B*
	

A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

- I would like to receive money if the ball drawn is black
- I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

I would like to bet on Urn A, and receive €15 if my bet is correct

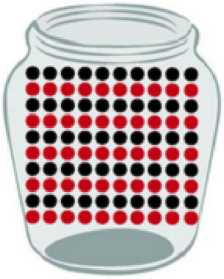
I would like to bet on Urn B, and receive €15 if my bet is correct

>>

Uncertain scenario γ

There are two urns with balls that can either be red or black.

- Urn A is composed of ***50 Black*** and ***50 Red*** balls.
- Urn B contains ***1000*** balls. The proportion of red and black balls is ***unknown***.

Urn A	Urn B
	
50 black 50 red	1000 balls % black ? % red ?

A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

- ○ I would like to receive money if the ball drawn is black
- ○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

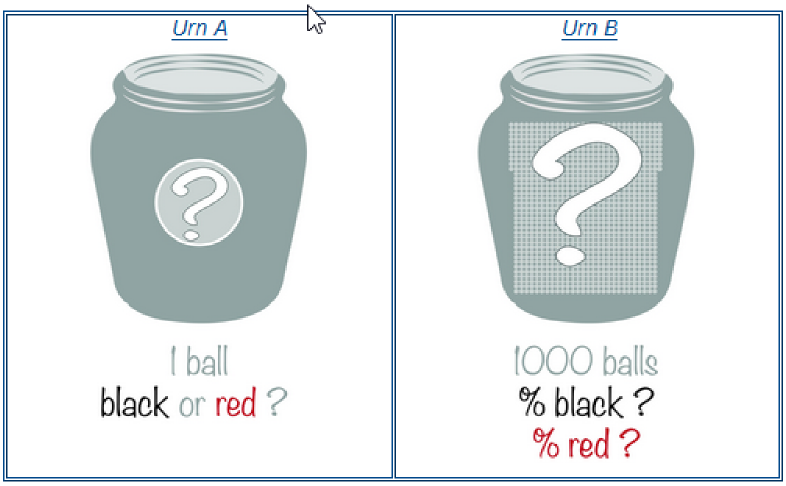
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Uncertain scenario κ

There are two urns with balls that can either be red or black.

- *Urn A contains **1** ball. The color of the ball is **unknown**.*
- *Urn B contains **1000** balls. The proportion of red and black balls is **unknown**.*



A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

○ I would like to receive money if the ball drawn is black

○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Certainty equivalents

Each of the 5 certainty equivalent scenarios is presented on two pages. Subjects first choose their winning color and then indicate their decisions in the choice list. The order in which the scenarios appear is randomized.

Page 1

Uncertain scenario u

*There is **1** ball in the urn. This ball is either black or red. The color of the ball is **unknown**.*

The ball will be drawn from the urn and its color will be observed.

Please select your winning color:

- ◯ I would like to have €15 if the ball drawn is black
- ◯ I would like to have €15 if the ball drawn is red

>>

Page 2

The ball in the urn is either black or red. The color of the ball is unknown.

Please indicate your decision on each line by marking the circle next to the option.

Option 1	Option 2
Have €15 if the ball drawn is black	Have €15 for sure
Have €15 if the ball drawn is black	Have €14 for sure
Have €15 if the ball drawn is black	Have €13 for sure
Have €15 if the ball drawn is black	Have €12 for sure
Have €15 if the ball drawn is black	Have €11 for sure
Have €15 if the ball drawn is black	Have €10 for sure
Have €15 if the ball drawn is black	Have €9 for sure
Have €15 if the ball drawn is black	Have €8 for sure
Have €15 if the ball drawn is black	Have €7 for sure
Have €15 if the ball drawn is black	Have €6 for sure
Have €15 if the ball drawn is black	Have €5 for sure
Have €15 if the ball drawn is black	Have €4 for sure
Have €15 if the ball drawn is black	Have €3 for sure
Have €15 if the ball drawn is black	Have €2 for sure
Have €15 if the ball drawn is black	Have €1 for sure
Have €15 if the ball drawn is black	Have €0 for sure

>>

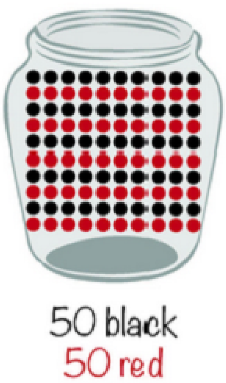
Uncertain scenario $\mathcal{E}$

*There are **100** balls in the urn. Each of the 100 balls in the urn is either black or red. The proportion of red and black balls is **unknown**.*

A ball will be randomly drawn from the urn and its color will be observed.

Uncertain scenario ρ

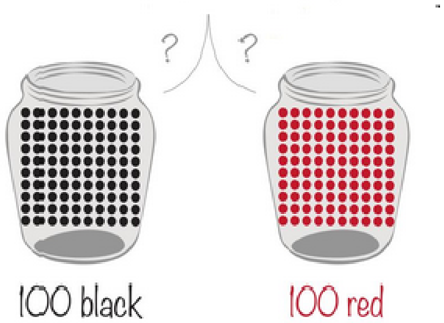
*The urn is composed of **50 Red** and **50 Black** balls.*



A ball will be randomly drawn from the urn and its color will be observed.

Uncertain scenario σ

*There are **100** balls in the urn. The 100 balls in the urn are either **all black** or **all red**. The exact composition is **unknown**.*



A ball will be randomly drawn from the urn and its color will be observed.

Uncertain scenario μ

*There are **1000** balls in the urn. Each of the 1000 balls in the urn is either black or red. The proportion of red and black balls is **unknown**.*

A ball will be randomly drawn from the urn and its color will be observed.

S2 Order Effects

In Table S2.1, we report the results of the Fisher exact tests comparing the choices made when the CE elicitation part appeared first and when the binary choices appeared first. None of the tests is significantat 5%, suggesting that choices made are not associated with the order of the task appearance.

Table S2.1: Attitudes measured with Certainty Equivalents

Pairwise comparisons	R vs. $E1$	R vs. $E100$	R vs. $E1000$	R vs. $P100$	$E1$ vs. $E100$	$E1$ vs. $E1000$	$E100$ vs. $E1000$	$E1$ vs. $P100$	$E100$ vs. $P100$
p-value	0.413	0.152	0.461	0.296	0.443	0.070	0.109	0.942	0.276

S3 Certainty Equivalents data

A second set of questions enables us to elicit the certainty equivalents (CEs) of the five uncertain situations. We used the following price-list design: for each uncertain situation, subjects made 16 binary choices between the prospect of receiving €15 if their bet was correct and receiving a sure amount of money ranging from €15 to €0 (with a decrease of €1 between each choice). Subjects were expected to choose the sure amount when it was higher than the CE of the uncertain situation and to switch to preferring the bet as the sure amount decreased to the point that is lower than the CE for the situation. We then use the switch point to compute the CE: it corresponds to the midpoint of the indifference interval implied by the switch point in that situation. Switching in the middle of the list implies a CE equal to the expected payoff (€7.5). The order of the uncertain situations R, $P100$, $E100$, $E1$, and $E1000$ was randomized.

Table **??** presents ambiguity attitudes measured with the CEs. Comparing the results with those obtained in Table 2 , we note that the share of ambiguity neutral in Part I (two sample proportion tests, ambiguity attitudes: measured with $E1$, p=0.391; measured with $E100$, p=0.015; measured with $E1000$, p=0.044) and indifferent in Part II is higher (two sample proportion tests, size preferences: between $E1$ and $E100$, p<0.001; between $E1$ and $E1000$, p=0.030; between $E100$ and $E1000$, p=0.015). It can be explained by the lack of precision of CE (that goes in step of €1) and does not allow to detect weak preferences.

Table S3.1: Ambiguity Attitudes measured with Certainty Equivalents

	Size of the ambiguous urn		
	$E1$ ($N = 83$)	$E100$ ($N = 83$)	$E1000$ ($N = 83$)
Ambiguity Aversion	26 (31.3%)	35 (42.2%)	33 (41.25%)
Ambiguity Neutrality	47 (56.6%)	41 (49.4%)	37 (46.25%)
Ambiguity Seeking	10 (12.1%)	7 (8.4%)	10 (12.5%)

Table S3.2: Size Preferences with Certainty Equivalents

	Size of the urns		
	$E1$ vs. $E100$ ($N = 83$)	$E1$ vs. $E1000$ ($N = 80$)	$E100$ vs. $E1000$ ($N = 81$)
Prefer Larger Urn	19 (22.9%)	21 (26.5%)	18 (22.2%)
Indifferent	41 (49.4%)	38 (47.5%)	52 (64.2%)
Prefer Smaller Urn	23 (27.7%)	21 (26.5%)	11 (13.6%)