Optimal Eligibility for Unemployment Insurance*

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Abstract

A minimum employment history is usually an eligibility condition to receive unemployment benefits. This paper characterizes its optimal level when unemployment risks are heterogeneous. First, by modeling the trajectory of workers on the labor market, I show that the optimal requirement follows a selection versus moral hazard trade-off. Second, thanks to french administrative data, I identify the behavioral responses to a requirement variation by using a bunching method and a regression kink design. These statistics are sufficient to assess the welfare implications of an eligibility change and to bound the optimal requirement. An eligibility reform has heterogeneous welfare implications within the labor force and its overall effect depends on the risk distribution and on frictions. The optimal criterion is below 4 months.

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1 Introduction

A new unemployed worker is asked to justify a minimum period of past employment to receive an unemployment benefit. In other words, the provision of an unemployment benefit is conditioned to a minimum employment history, i.e a number of days worked before the claiming date. In 2019, this requirement amounts to 12 months in Germany or Spain, 6 months in Netherlands and 4 months in France over a given reference period[1] (Asenjo and Pignatti, 2019).

Its implementation is based on a theoretical paper by Hopenhayn and Nicolini (2009) in which they show that it's optimal to condition the benefits paid to unemployed workers on their employment history if quits cannot be distinguished from layoffs. Indeed, in addition to moral hazard in unemployment, unemployment insurance (UI) provides incentives either to employees to decrease their labor supply, either to employers to adapt their labor demand (Albanese et al. (2020)). On the supply side, some workers can strategically behave by resigning to obtain unemployment benefits, accepting a short-term job to renew their unemployment rights or exerting less effort at work. On the demand side, some employers can also strategically behave by proposing a specific contract duration or firing covered workers. Several empirical works have documented this moral hazard in employment. For instance, the transition from employment to compensated unemployment changes with eligibility in several countries (Baker and Rea, 1998 for Canada, Rebollo-Sanz, 2012 for Spain, Martins, 2016 for Portugal or Khoury et al., 2020 for France). Some other strategic behaviors are detected, such that early retirement (Baguelin and Remillon, 2014) or contract manipulation to benefit from UI extension (Khoury, 2019). This problem cannot be totally tackled by excluding resigners from compensation or by regulating the duration of contracts as voluntary quits are not completely observable and as employers and employees cooperate to choose the contract termination. Therefore, the government manipulates this minimum employment history requirement to provide incentives to keep jobs longer. It's a policy instrument used to fight against the moral hazard in employment as the replacement rate or the potential benefit duration (PBD) are used to combat the moral hazard in unemployment.

Nevertheless, eligibility is not a costless solution. Workers who loose their job without fulfilling the minimum requirement are excluded from the system and experience a significant income loss. Within the workforce, the risk of being unemployed is very heterogeneous since being unemployed at a given period depends on several personal characteristics, on the behavior of firms and on the macroeconomic situation. Each worker faces a different layoff probability and a different job finding ability. Therefore, implementing an eligibility function reduces the moral hazard in employment but

[1] The reference period also changes across countries. It corresponds to the last 30 months in Germany, 72 months in Spain, 9 months in Netherlands and 24 months in France.

disqualifies workers who are unable to work longer, those whose unemployment risk is high. Since manipulating the minimum employment history criterion generates benefits and costs, one criterion value may be socially preferable. This paper attempts to answer the following questions: What is the trade-off behind the choice of the criterion? What would be the welfare implications of a requirement variation within the labor force? What is its optimal level?

To answer these questions, I model the behavior of workers who are temporarily employed in unstable jobs on a labor market where unemployment benefits depend on their work history. The eligibility function is defined as the link between the number of days worked before the job loss and the level of unemployment benefits. An unemployment risk is assigned to each worker according to his ability to keep an unstable job or to exit unemployment. This heterogeneity is key to understand the different implications of eligibility within the labor force since it generates disparities in affiliation and unemployment spell. Several personal factors can explain this risk such that age, gender, sector or unobserved ability. It also takes into account the vacancies they face, i.e the choices made by firms on the demand side of the labor market. I also consider frictions such that hurdles which prevent them to manipulate their employment history, for instance imperfect information or contract rigidity. The model allows to derive a formula *à la* Baily-Chetty which describes the trade-off faced by the social planner when it implements an eligibility process. Then, I isolate the effect of a variation in the minimum employment history requirement on different risk groups and on the whole economy. I connect my theory to data by exhibiting statistics that are sufficient to assess the welfare implications of such a variation. To identify these statistics, I use the FH-DADS, a french administrative data set which allows to follow $\frac{1}{12}^{th}$ of the workforce in employment and in unemployment over 10 years. I exploit the incentives to bunch provided by the discontinuity in the eligibility function to identify the behavioral responses in terms of employment duration. I take advantage from a kink in the relationship between the benefit and past wages to estimate the behavioral responses in terms of unemployment duration. Finally, I plug my results into the trade-off formula to quantify the welfare effects created by a marginal requirement variation. Since I observe two different thresholds in my data (4 and 6 months), I can bound the value of the optimal criterion.

The model shows that, on the one hand, increasing the eligibility threshold provides incentives locally. Only a minority of workers bear the cost of the reform and generate its benefits by responding. This minority corresponds to the most risky individuals, those who become unemployed with exactly the required affiliation. More precisely, a higher requirement excludes them from compensation. In reaction, they can either try to increase their affiliation to catch up with the new requirement or decrease their unemployment duration to suffer less. These behaviors alleviate the budget constraint and allow a tax cut. The total decrease in taxation rises the welfare of employment spells of all workers: the minority shares the benefits of their extra efforts with the majority. Therefore, the optimal minimum employment history requirement is characterized by a exclusion versus moral hazard in employment trade-off. It differs from the usual insurance versus moral hazard in unemployment trade-off characterizing the optimal replacement rate. This overall trade-off hide large disparities in welfare implication. The excluded experience a relative strong decrease in well-being. The magnitude of their welfare decline depends on their ability to increase their affiliation or to decrease their unemployment duration. The welfare of other workers, which are in the majority, is enhanced by the tiny tax cut since only the minority changes their behaviors.

The propensity of impacted workers to meet the new requirement depends mainly on their intrinsic capacity to manipulate their employment duration. This ability relies on their unemployment risk and on frictions. A low risk worker is able to keep a job longer whereas a high risk worker is likely to loose rapidly his job even if he exerts more efforts at work. Frictions prevent some workers who have the ability to manipulate their employment duration to fulfill the eligibility criterion. Consequently, increasing the threshold excludes those who are too risky and those who are constrained by frictions. The rest of impacted individuals manage to catch up with the new requirement by exerting an extra effort at work.

Therefore, the overall welfare effect of a reform depends on two questions: who are directly impacted? and are they able to react? If manipulating the criterion generates low reactions in employment, the usefulness of this policy instrument can be questioned. However, if reactions are high, increasing the criterion allows to increase the employment spells without excluding. In this case, the policy instrument would be efficient. To quantify the overall and particular welfare implications of such a reform, I need to estimate the reaction capacity of impacted workers in order to spot those who would be unable to meet the new requirement and identify how they react in unemployment to the loss of their unemployment benefits.

To do so, I follow the bunching literature (Saez, 2010, Kleven and Waseem, 2013) by exploiting the notche in the budget set created by the discontinuity at the minimum requirement in the eligibility function. Having worked more than the threshold increases discontinuously the income as the worker becomes eligible, it provides incentives to those who become unemployed with an employment history just below the threshold to work a little bit more. By assuming that the natural employment history, i.e the employment duration before a job loss which is not impacted by the incentives provided by the notche, is a proxy for the unemployment risk, I can designate the type of workers who are excluded and compute their shares in the whole population. I find that the 182 days (resp. 122 days)[2] requirement provides incentives to increase the employment history by up to 12 days (6

[2]it's equivalent to 6 months (resp. 4 months)

days). The level of frictions seems to be high around 80%. Therefore, a one day increase in the 6 months (resp. 4 months) requirement excludes 20% of workers who naturally choose to work 170 days (116 days) and 80% of those who naturally choose to work exactly 182 days (122 days). It finally impacts directly around 1% of the whole population. Then, I identify their reactions in terms of unemployment duration by using a kink observed in the relationship between the daily benefit and the daily reference wage. I find that the elasticity of the mean unemployment duration with respect to the benefit level is 0.44. For instance, it translates into an increase of 35% in the expected unemployment duration at the 6 months threshold. By plugging those estimates into my model, I show that, for reasonable values of marginal utility in unemployment and in employment, a marginal increase in the threshold at 6 and 4 months decreases the utilitarian total welfare in the economy: the optimal criterion is inferior to 4 months. The minimum employment history requirement is an inefficient policy instrument since the incentives provided are too weak and too local to encompass the cost of excluding the workers who value public insurance the most.

The main contribution of this paper is to highlight the trade-off behind the optimal eligibility requirement and the implications of a criterion variation within an heterogeneous labor force. Only few papers address theoretically the question of eligibility. Hopenhayn and Nicolini (2009) justify the implementation of an eligibility requirement when layoff are not observable. Ortega and Rioux (2010) emphasize that the criterion can support job creation since, after the exhaustion of their unemployment rights, workers are less demanding on the quality of their job to obtain new unemployment rights. Andersen et al. (2018) compares the incentives of eligibility instruments with generosity instruments by allowing for endogenous search in a search and matching model. They show that requirement can provide incentives for job search as the benefit duration. But no paper writes down a formula *à la* Baily-Chetty (Baily, 1978, Chetty, 2006). I also contribute to the evaluation of eligibility reforms by using a sufficient statistics approach, as done in Chetty (2008) for the replacement rate, to assess the welfare implications of a requirement variation. Empirically, I also provide new estimations of worker's reactions to a change in eligibility. I identify the reactions in terms of affiliation as Khoury et al. (2020) and in terms of unemployment duration as Landais (2015).

The paper is organized as follows: in section 2, I present a model and describe the theoretical welfare implications of an eligibility variation. In section 3, I present the data and identify my sufficient statistics. In section 4, I assess the welfare implications of an eligibility variation. Section 5 is the general conclusion.

2 Optimal theoretical eligibility

I firstly model the trajectory of workers over their life cycles on a labor market where an unemployment benefit is provided subject to an employment history requirement. Workers are heterogeneous in their unemployment risk and can strategically behave as employee or as unemployed. The presented model is in the spirit of Browning et al. (2007) and allows to derive the trade-off characterizing the optimal minimum employment history required.

2.1 The model

I focus on workers who experience unstable jobs in their working life. Time t is continuous and discounted at rate r. Professional life starts at 0, lasts indefinitely and is split into 3 periods: unstable job, unemployment and stable job. Workers have access to a risk free asset A and enjoy their consumption c through a concave utility function u(.) $(u'(.) > 0,\ u''(.) < 0)$. They are heterogeneous in their unemployment risk noted α. This risk can be seen as their ability to keep unstable jobs or search efficiently for a stable job. A high ability to remain employed in an unstable job is a sign of a low employment risk whereas a low capacity to find a new job corresponds to a high unemployment risk. Therefore, the cost paid to reach a given duration in an unstable job or the cost paid to exert a given search effort are different between types. The probability density function (pdf) of types is noted $a(\alpha)$ and defined on the support $[0, \infty[$, where $\mathcal{A}(\alpha)$ is the cumulative distribution function (cdf).

Firstly, at $t = 0$, a worker of type α is hired in an unstable job. His position is unstable in the sense that it will be destroyed, the worker has heard of the fragility of his position, the firm told him that his position won't last indefinitely. Nevertheless, the worker can impact the duration of his position by exerting efforts to keep the position open[3]. The position holder chooses the job duration h which generates a cost noted $\psi^e_\alpha(h)$. This cost depends on his risk α and is increasing and convex in the job duration $({\psi^e_\alpha}'(.) > 0,\ {\psi^e_\alpha}''(.) > 0)$. Secondly, once the unstable job is destroyed at $t = h$, he falls into unemployment. By exerting efforts to search for a new job, the worker controls his expected unemployment duration d by choosing the job finding rate λ[4]. The cost of search depends on worker's risk and is increasing and convex in the job finding rate: $\psi_{u,\alpha}(\lambda)$ $({\psi^u_\alpha}'(.) > 0,$ ${\psi^u_\alpha}''(.) > 0)$. Finally, he is hired in a stable job and works indefinitely.

Working yields a wage w on which a tax τ is levied to fund an UI system which provides a benefit

[3] for instance, those efforts could correspond to a better negotiation in order to obtain a renewal of a fixed-term contract or a better work to show to the firm the importance of the position

[4] The probability to exit unemployment follows an exponential law of parameter λ. In average, workers who choose λ stay unemployed for a period $\frac{1}{\lambda}$. Therefore, given an unemployment risk, the average unemployment duration (noted d) is $d = \frac{1}{\lambda}$.

to unemployed workers. The system conditions the provision of a benefit on the employment history of each worker. This employment history corresponds to the affiliation of each new unemployed person. If the worker has worked enough, i.e $h \geq \theta$, he receives an unemployment benefit. If he does not fulfill the minimum requirement θ, i.e $h < \theta$, he receives a lower benefit such that the minimum social income. Therefore, the unemployment benefit depends on the comparison between the employment history h and the minimum requirement θ: $b(h;\theta)$. The form of the benefit function is described in appendix A.

The present value of being employed in an unstable job (noted $V_\alpha(A_0)$) is equal to, given the consumption choices $\underline{c_t}$ and the chosen employment duration h, the discounted utility generated by the consumption of the net wage earned in the unstable job minus the cost of the exerted effort, plus the value of being unemployed at h:

$$\begin{aligned}
V_\alpha(A_0) = \max_{\underline{c_t},h} \int_0^h e^{-rs} u(\underline{c_s})\mathrm{d}s + e^{-rh} U_\alpha(A_h, h) - \psi_\alpha^e(h) \\
\text{s.t } A_0 + \int_0^h e^{-rs}(w - \tau - \underline{c_s})\mathrm{d}s - e^{-rh} A_h \geq 0 \\
e^{-rh} A_h \geq 0 \\
\text{where } A_s = e^{rs}\left(A_0 - \int_0^s (w - \tau - \underline{c_t}) e^{-rt} dt\right)
\end{aligned}$$

The present value of being unemployed (noted $U_\alpha(A_0, h)$) corresponds to, given his consumption choices c_t and his job finding rate λ, the consumption in unemployment given the probability to remain unemployed at each period plus the value of being employed given the probability to find a job at each period minus the searching cost. The flow of unemployment incomes depends on h and λ. At each period with a probability λ, he finds a stable job for eternity. This job yields a constant consumption $\bar{c} = \frac{u(rA_s + w - \tau)}{r}$ (proof in appendix A). The value function is written as follows:

$$\begin{aligned}
U_\alpha(A_0, h) = \max_{c_t,\lambda} \int_0^\infty u(c_t) e^{-(r+\lambda)t} dt + \lambda \int_0^\infty \frac{u(rA_s + w - \tau)}{r} e^{-(r+\lambda)s}\mathrm{d}s - \psi_\alpha^u(\lambda) \\
\text{s.t } A_0 + \int_0^\infty e^{-rt}(b_t(h;\theta) - c_t) dt \geq 0 \\
\lim_{s\to\infty} e^{-rs} A_s \geq 0 \\
\text{where } A_s = e^{rs}\left(A_h - \int_0^s e^{-rt}(b_t(h;\theta) - c_t) dt\right)
\end{aligned}$$

By defining μ_{BC} and μ_{CC} as the Lagrange multipliers of respectively the budget constraint and the compatibility constraint, the first order conditions for consumption are:

$$\underline{c_t} : u'(\underline{c_t}) = \lambda \int_0^\infty e^{-\lambda s} u'(rA_s + w - \tau)\mathrm{d}s + \mu_{BC} \tag{1}$$

$$c_t : u'(c_t) = e^{\lambda t} \lambda \int_t^\infty e^{-\lambda s} u'(rA_s + w - \tau)\mathrm{d}s + \mu_{BC} \tag{2}$$

$$\dot{c}_t = \frac{\lambda(u'(c_t) - u'(rA_t + w - \tau))}{u''(c_t)} \tag{3}$$

$$\dot{A}_t = rA_t + b_t(h;\theta) - c_t \tag{4}$$

Consumption in the unstable job is constant overtime since it does not depend on t: $\underline{c_t} = \underline{c}$. In unemployment, consumption and saving decrease over the spell according to the laws of motion 3 and 4. The agent depletes his assets to compensate the income gap between the net wage and the unemployment benefit. Next, at the beginning of their active life, each worker α optimizes by selecting the employment duration h which equalizes the marginal cost of exerted effort to the benefit of earning more wage, accumulating more unemployment rights and potentially saving more. Therefore, his choice mainly depends on the shape of his cost function (i.e his unemployment risk) and on the shape of the eligibility function. The optimal employment duration h follows:

$$\psi_\alpha^{e\,\prime}(h) = e^{-rh}\left(u(\underline{c}) - rU_\alpha(A_h, h) + \frac{\partial U_\alpha(A_h, h)}{\partial h} + \frac{\partial U_\alpha(A_h, h)}{\partial A_h}\dot{A}_h\right) \tag{5}$$

Once the worker is unemployed, he selects a job finding rate by equalizing the marginal cost of searching more intensively to the marginal gain of being unemployed which depends on the unemployment rights previously accumulated. Therefore, his choice depends mainly on the shape of his cost function (i.e his unemployment risk) and on his employment history. The optimal job finding rate λ follows:

$$\psi_\alpha^{u\,\prime}(\lambda) = \int_0^\infty e^{-(r+\lambda)s}(-su(c_s) + (1 - \lambda s)u(rA_s + w - \tau))\mathrm{d}s \tag{6}$$

Since choosing exactly his affiliation is a strong assumption, I assume that it exists frictions which prevent the worker to perfectly optimize his employment duration. Each worker is randomly constrained by frictions with an exogenous probability noted ϕ. Those frictions can correspond to different hurdles observed in reality. For instance, a lack of information about UI rules, a rigidity in labor demand or an exogenous job destruction. Constrained workers do not internalize the effect of their behavior on the benefits they get once they become unemployed. In other words, their choice in employment duration ignores the incentives given by the discontinuity in the eligibility function. Therefore, they equalize only the marginal cost of exerting effort to get a longer duration to the benefit of earning more labor income. First order conditions 5 and 6 show that the employment duration and the average unemployment duration are mainly driven by the unemployment risk. I assume that each unemployment risk corresponds to a different natural employment duration, the employment duration chosen when the incentives provided by the unemployment insurance system are ignored. Hence, I assume that each α yields a different natural employment duration and takes its value. In other words, the unemployment risk is expressed as the natural duration in unstable jobs. High risk workers exhibit a low α whereas low risk workers a high α. Only when the unemployment insurance system provides incentives, the effectively chosen employment duration differs. The natural employment duration is the solution of the first order condition of constrained workers: $\psi_\alpha^{e\prime}(\alpha) = e^{-r\alpha}\left(u(\underline{c}) - rU_\alpha(A_\alpha, \alpha) + \frac{\partial U_\alpha(A_\alpha,\alpha)}{\partial A_\alpha}\dot{A}_\alpha\right)$ To sum up, for each type α, a share ϕ makes the constrained choice α and a share $1-\phi$ chooses h. If types α are not subjected to the incentives given by a discontinuity in the eligibility function, the constrained choice is equivalent to the optimal one, i.e $h = \alpha$ if $\frac{\partial b(h,\theta)}{\partial h}|_{h=\alpha} = 0$. Since frictions can impact the eligibility status of workers within a group α, I denote $\tilde{\lambda}$ as the job finding rate chosen by constrained workers. In the following, I denote the unconstrained choices of individuals α as h_α and λ_α and those of constrained workers as α and $\tilde{\lambda}_\alpha$. The pdf of the observed distribution of affiliation, given frictions and the heterogeneity in unemployment risk, is noted f.

2.2 Optimal instrument

The social planner has three possible policy instruments: the tax, the unemployment benefit or the minimum employment history requirement. I assume that the unemployment benefit is fixed. In this subsection, I derive the trade-off which characterizes the choice of the threshold from the point of view of the social planner in different contexts. I firstly assume that it exists only one unemployment risk. Secondly, the heterogeneity is taken into account. Thirdly, I add frictions. In these three contexts, the welfare of an individual α is measured as the sum of the discounted utility flows over life: $V_\alpha(A_0)$. The social planner is assumed to be utilitarian: he maximizes the sum of individual welfare in the economy given the constraint of a balanced budget. This budget constraint equalizes the benefits paid to all unemployment spells to the tax levied on all jobs, i.e the tax times the sum of discounted unstable jobs and stable jobs. Each program of the social planner and all the derivations are available in appendix A.

2.2.1 Homogeneous unemployment risk without frictions

I firstly assume that the unemployment risk is homogeneous. It can also be seen as an actuarially fair UI, where the social planner discriminates each unemployment risk by setting a different threshold and a different tax. The optimal minimum requirement θ is obtained by equalizing the first order condition to zero:

$$CS_\alpha = -\frac{\partial V_\alpha}{\partial \tau} FE_\alpha \tag{7}$$

where CS_α is the consumption smoothing variation experienced by workers α and FE_α corresponds to the fiscal externality created by the behavioral reactions of individuals α. Equation 7 states that the optimal threshold equalizes the consumption smoothing variation to the effect of the fiscal externalities on the employment periods.

Since there are no frictions and only one unemployment risk, each worker in the economy become unemployed with the same affiliation. A marginal increase in the threshold has an effect only if at least one worker become unemployed with an affiliation equal to the threshold. Therefore, the value of the optimal threshold depends on the unemployment risk present in the economy. If the considered unemployment risk leads to an affiliation above or widely below the threshold, increasing the threshold has zero effect. However, in the case where the natural affiliation of workers is equal to the threshold or just behind, an eligibility variation matters. Workers would loose their eligibility, their unemployment income would decrease. Since less benefits are paid, in consequence, their disposable income in employment would also increase. Those effects are described by a negative consumption smoothing variation on the left hand-side ($CS_\alpha < 0$). To regain their benefits, they would react by increasing their affiliation. The capacity to increase their employment duration depends on their labor market risk. If the unemployment risk present in the population leads to a natural affiliation equal to the threshold, the cost of exerting an extra effort is lower than the benefit of remaining compensated. They effectively catch up with the new requirement and the fiscal externality captures the effect on the tax of this increase in employment duration. However, if the considered unemployment risk is higher, workers could be unable to increase their affiliation and become effectively ineligible. In that case, the externality on the tax generated by the willingness to find a job rapidly in order to avoid the low benefit and to earn a living wage is captured by FE_α. In both cases, either increasing the affiliation or either rising the job finding rate, the fiscal externality is negative ($FE_\alpha < 0$).

The threshold which maximizes the welfare of the considered population is the threshold which excludes nobody but provides incentives to increase their employment duration. Therefore, discriminating workers by varying the threshold according to their unemployment risk can be efficient since it extracts the maximum effort from each worker without any exclusion. Nevertheless, in reality, it's impossible to observe precisely the unemployment risk of workers and this risk changes also over time and over life. So the social planner has to deal with an heterogeneity of risks.

2.2.2 Heterogeneous unemployment risk without frictions

I secondly assume that unemployment risks are heterogeneous within the labor force. Again, only workers on the threshold are excluded and try to react. Therefore, I need to know which labor market risks α are present on the threshold. Equation 5 states that the constrained employment duration is optimal when $\frac{\partial b(h;\theta)}{\partial h}|_{h=\alpha} = 0$. Based on the bunching literature, I assume that only workers who are naturally close but behind the threshold exert an extra effort to bunch on the threshold. The ability to bunch depends on the cost of increasing their employment duration, i.e on their unemployment risk. I define $\alpha = \theta - \Delta$ as the labor market risk of the marginal buncher, i.e the highest labor market risk observed on the threshold[5]. The distribution of affiliations f can be decomposed as follows:

$$f(x) = \begin{cases} a(x) & \text{if } x \notin [\theta - \Delta, \theta] \\ 0 & \text{if } x \in [\theta - \Delta, \theta[\\ \int_{\theta-\Delta}^{\theta} a(z)\mathrm{d}z & \text{if } x = \theta \end{cases}$$

To sum up, all workers naturally behind the threshold are not able to bunch. It's too costly for the most risky. Only a share of them bunch on the threshold. On the threshold, several types of workers are present: individuals whose unemployment risk is too high to be naturally eligible but who are able to fulfill the requirement by exerting more efforts ($\theta - \Delta \leq \alpha < \theta$) and those who are naturally present on the threshold ($\alpha = \theta$). Each worker on the threshold will be impacted by a marginal change in eligibility and will react. In case of a threshold increase, the reactions can be classified into two groups: the group 1 called the "marginal bunchers" composed by individuals whose unemployment risk is too high to be able to exert a second extra effort to bunch on the new requirement ($\alpha = \theta - \Delta$) and the group 2 called the "bunchers" composed by the others ($\alpha \in]\theta - \Delta; \theta]$). Group 2 manage to exert a new extra effort to meet the new requirement. I define a_i as the share of group i in total population. In this context, the trade-off is:

$$a_1 CS_1 + a_2 CS_2 + CS_{others} = -\int_0^{\infty} \frac{\partial V_\alpha}{\partial \tau} \mathrm{d}\mathcal{A}(\alpha)(a_1 FE_1 + a_2 FE_2) \tag{8}$$

The left hand-side describes the consumption smoothing variation experienced by different groups of workers. A marginal threshold increase excludes the mass $f(\theta) = a_1 + a_2$ from eligibility. Among those impacted workers, group 1 and 2 experience a loss in consumption smoothing due to the

[5] In other words, Δ is the maximum extra effort that workers want to exert to being eligible. It's closely related to the elasticity of affiliation with respect to compensation.

decrease in compensation. Their utility in unemployment drops. Without taking into account the reactions, the exclusions reduce the expenses of the system, so at the same time, allow the social planner to decrease the tax. This tax cut is welfare enhancing for everyone especially for the most risky since their marginal utility in employment is likely to be higher. The other workers, those whose compensation in unemployment is intact, only benefit from the tax cut. Therefore, CS_1 and CS_2 are negative whereas CS_{others} is positive but lower. The right hand-side describes how impacted workers react and how it affects the utility of all employees. Group 1 are too risky to be able to keep their unstable job up to the threshold. They come back to their natural employment duration. Being uncompensated, they reduce their average unemployment duration by searching more intensively for a job. FE_1 describes how the combination of the employment duration and the average unemployment duration decreases affects the tax. Its sign depends on the magnitude of the two effects. Group 2 are able to exert an other extra effort to remain compensated. They increase their employment duration proportionally to the threshold variation. Their average unemployment duration does not change since their situation in unemployment after having reacted is constant. FE_2 is negative and describes how the rise in employment duration affects the tax. The reactions of impacted workers generate externalities on the tax levied on employees' wage. It results a variation of the utility of all employees captured by $(-\int_0^\infty \frac{\partial V_\alpha}{\partial \tau} \mathrm{d}\mathcal{A}(\alpha))$.

If the left hand-side is higher than the right hand-side, increasing the threshold generates more welfare gains for the employees due to the tax cut than losses in consumption in unemployment for the excluded. The reform would be then welfare enhancing. In reality, workers are not able to manipulate exactly their affiliation. Frictions must be taken into account.

2.2.3 Heterogeneous unemployment risk with frictions

I thirdly assume that it exists an heterogeneity in unemployment risk and that workers can be prevented from exactly choosing their employment duration. Those frictions only change the distribution of affiliations around the discontinuity and create a new way of reacting to a threshold increase. Regarding the distribution of affiliations, less workers manage to bunch, a significant part of them are stuck at their natural location. The amount of bunchers on the threshold is therefore reduced. The distribution of affiliations f is different:

$$f(x) = \begin{cases} a(x) & \text{if } x \notin [\theta - \Delta, \theta] \\ \phi a(x) & \text{if } x \in [\theta - \Delta, \theta[\\ (1-\phi)\int_{\theta-\Delta}^{\theta} a(z)\mathrm{d}z + \phi a(\theta) & \text{if } x = \theta \end{cases}$$

Regarding the reactions in case of an eligibility change, there is a new group of workers who react differently. I define a group 3, called the "stuck", which are composed by those who were naturally on the previous threshold but who are constrained and are unable to exert an extra effort to fulfill the new one. The three types of reactions among impacted workers are summarized by figure 1. In this context, the overall trade-off is:

$$a_1 CS_1 + a_2 CS_2 + a_3 CS_3 + CS_{others} = -\frac{\partial W}{\partial \tau}(a_1 FE_1 + a_2 FE_2 + a_3 FE_3) \tag{9}$$

Again, the left hand-side describes the variation in utility experienced in unemployment by the excluded minority and the variation in utility experienced in employment due to the tax variation created by this exclusion. Excluded workers have an affiliation equal to the previous threshold: $f(\theta) = a_1 + a_2 + a_3$. Their well being decreases because the negative effect of being excluded surpasses the positive effect of the tax cut. Since for the rest of the population nothing change in unemployment they only enjoy the tax cut. Their well being increases but little. Again, the right hand-side describes the variation in utility experienced in employment by the whole population due to the tax variation generated by the reactions of impacted workers. Groups 1 and 2 have the same reactions as in the previous context when frictions were omitted. Group 3 is constrained by frictions. Their employment duration is equal to the threshold but they cannot increase it. Therefore, they try to exit unemployment by increasing their job finding rate. FE_3 is negative and describes how the change in the job finding rate of group 3 affects the tax. Optimal θ equalizes the effect of insurance selection to fiscal externality.

The welfare effect is very heterogeneous within the labor force. A majority is not impacted by the reform and benefits from the tax cut. Among impacted workers, those who are able to respond have only to exert more effort in employment to fulfill the new requirement. Those who are unable to respond, either because they have a high unemployment risk or because they are constrained by frictions, experience a high consumption drop during their unemployment spell. This misadventure is either amplified or diminished according to their ability to find a new job rapidly. An important point is that these benefits created by their behavioral responses are shared by the entire population. In other words, affected workers relax the budget constraint for all. So, in addition to paying the costs, they share the benefits they generate. Therefore, the sign of the overall welfare change due to a requirement variation depends on two questions: among impacted workers, who is unable to fulfill the new requirement? Are they also unable to decrease their unemployment duration? The answer of these two questions depends on the mass of impacted workers, i.e the distribution of unemployment risks and frictions, and on behavioral responses. Finally, even if with the reform the sum of individual well-being would increase, it hides a strong heterogeneity of situations: the welfare of some high risk workers goes tragically down.

To quantify the welfare implications of a requirement variation, I need to count the number of workers contained in each of the three groups and identify their corresponding behavioral responses. To recover the distribution of unemployment risks, I construct a counterfactual distribution of employment durations, i.e a distribution of employment durations without any incentives provided by the discontinuity in the eligibility function. Then, by using this counterfactual, I can estimate two statistics Δ and ϕ that are sufficient to identify the behavioral responses in employment. Finally, I identify locally the elasticity of the average unemployment duration with respect to the benefit level of workers. This last statistic allows to calibrate the change in job finding rate generated by workers who are *in fine* excluded.

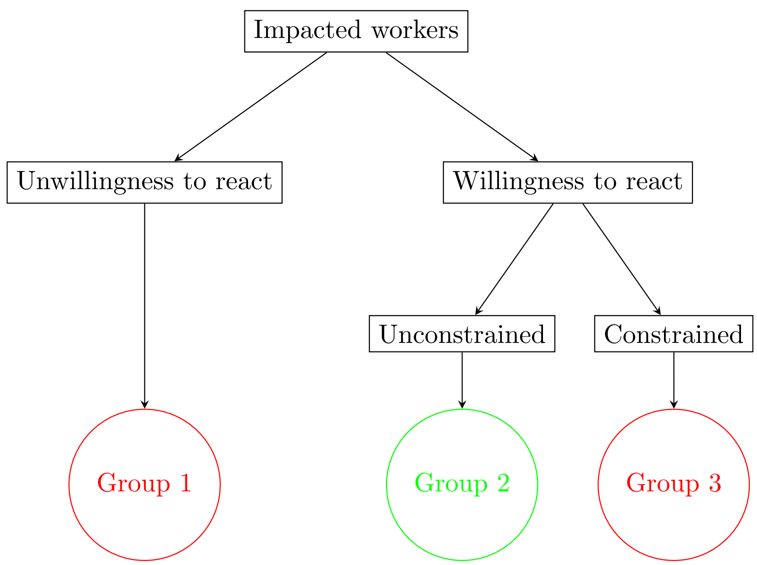


Figure 1: Different groups according to their reaction within impacted workers

Note: The willingness to react depends on the unemployment risk of each worker whereas the constraints on reactions depends on frictions. A worker who is unwilling to meet the new reaction has to pay too much cost to exert an extra effort. The groups in red are *in fine* ineligible whereas the group in green is *in fine* eligible.

3 Linking theory to data: estimation of 3 statistics

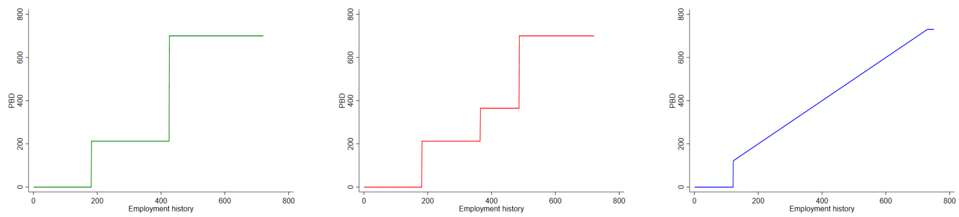
In this section, I connect my model to data by exhibiting statistics that are sufficient to perform a welfare analysis on eligibility reforms. I use french administrative data to construct the joint distribution of employment durations and unemployment durations. Then, by using the notche created by a discontinuity in the eligibility function, I identify the reactions of each type of workers to a shift in the eligibility threshold.

3.1 The data

The FH-DADS are panel data which cover the employment and unemployment periods of $\frac{1}{12}^{th}$ of the french population from 2003 to 2012. They match two datasets: the "Fichier Historique", a historical database from the french national employment agency (FH) to the "Déclarations Annuelles de données sociales" (DADS). It allows to follow workers over 10 years throughout their employment and unemployment periods. In addition to some personal characteristics, the data contain information on employment contracts and employers on the employment side and information on benefits received and unemployment durations on the unemployment side. For each observed unemployment spell, it's possible to compute the unemployment duration and the attached employment history or affiliation.

The employment agency provides several types of allocation according to the situation of the claimant. I select spells corresponding to the provision of the unemployment benefit called ARE, i.e a benefit provided to unemployed workers according to their employment history and funded by contributions paid by employees and employers. Then I exclude each worker that are entitled to special eligibility or benefit rules. Therefore, workers either aged more than 50, either previously employed as temporary worker or in the culture and art industry are dropped. I remove also individuals who are employed by a private individual one time in their working life because private employers are only observed after 2009. Finally, under some conditions, UI rules allow workers to continue to receive a past benefit if there is a remainder from a previous compensated unemployment spell. The corresponding observations are excluded because the employment history does not correspond to the correct PBD, since the unemployment rights have already been partly used in a past unemployment spell.

I define as employment history, or affiliation, the number of days worked within 28 months before an unemployment spell. I define as an exit from unemployment the first day worked once the worker is no longer registered as job seeker at the employment agency. However, FH data do not provide information on uncompensated unemployment spells. These types of periods can correspond to either an eligible worker who did not claim his rights, either a worker who exited the labor force or someone who did not work enough to become eligible. Because in my model, eligible unemployed workers always claim their benefit and cannot exit the labor force, I'm only interested in computing the employment history of each unobserved unemployment spells of workers who did not work enough to become eligible. I use the DADS data to compute these missing affiliations. I assume that each period of two weeks without any job corresponds to an unemployment spell. I do not take into account shorter periods because they are likely to describe a job to job transition. At the end, I end up with a sample of 2 949 575 unemployment spells for 1 344 393 individuals.



(a) 2004 (b) 2006 (c) 2009

Figure 2: Observed eligibility functions

Note: Under the 2004 convention, the eligibility function has two steps at 6 and 14 months. Under the 2006 convention, there are three steps at 6, 12 and 16 months. Under the 2009 convention, from 4 to 24 months the eligibility function follows the "one day worked, one day paid" rule.

In France, the UI rules are renegotiated every 2 or 3 years by unions and firm representative. The final agreement is a convention in place until the next reform or negotiation. I observe in my sample three different conventions defining for each of them a different eligibility function. The convention 2004 established an eligibility function containing two possible PBD between January 2004 and January 2006. The minimal requirement is having worked at least 6 months. If your employment history is above 14 months, your benefit duration is 700 days otherwise it would be only 213 days (see figure 2a). Then, between January 2006 and April 2009, the convention 2006 added one more possibility: if your employment history is between 6 months and 1 year your benefit duration is 213 days, between 1 year and 16 months gives you 365 days of benefit and more than 16 months yields a duration of 700 days (see figure 2b). The shape of the eligibility function has been reformed in April 2009. The minimal requirement decreased to 4 months of work and one more day worked increases your PBD by the same amount up to a maximum of 730 days (see figure 2c).

In the following, I focus on the minimum requirement of 6 months observed under the 2006 convention and 4 months observed under the 2009 convention. The 2004 convention is used to test the robustness of my results. To assess the welfare implications of a change in the minimum requirement, I need the distribution of employment and unemployment durations. To follow my model, I define d, the duration of unemployment, as the difference between the ending date of the previous employment contract and the last day registered at the employment agency (or the first day worked for ineligible workers). For my robustness checks, I consider other definitions for d[6]. I plot the distributions of unemployment durations for each affiliation on figure 9 in appendix B.

[6] I define also: d_1 as the duration of registered unemployment (registration dates to the unemployment agency),d_2 as the duration of claimed unemployment (from registration to first day worked) and d_3 the duration of total non-employment (from job loss to first day worked).

Ineligible workers seem to mainly exit unemployment within the five first months of unemployment. Eligible workers increase their effort when they get closer to the exhaustion of their rights, i.e closer to the red line. They are also likely to exit unemployment within the first five months of their spell. A better representation of choices in employment duration is to plot the distribution f as on figure 10 in appendix B. On each discontinuity in the eligibility function, there is an excess mass of workers who seem to bunch to enjoy a higher PBD during unemployment. By subtracting the distribution of affiliations under the conventions 2006 and 2009, we can see that the choices of h are similar except around each discontinuity (see figure 3). These discontinuities provide incentives to employees to keep their job longer. That's why, some workers bunch on thresholds to increase their potential benefit duration. In my model, the choice in job duration depends on the worker's unemployment risk α and on frictions (see equation 5). Each worker within the same type α chooses the same duration, except those who are unconstrained and impacted by a discontinuity. The distribution of unemployment risks α is assumed to be smooth and to generate the observed distribution of affiliations due to the discontinuities in the eligibility function. Therefore, each h corresponds to a different α except on discontinuities where workers with a lower α can bunch and be mixed with the type α naturally located on this discontinuity.

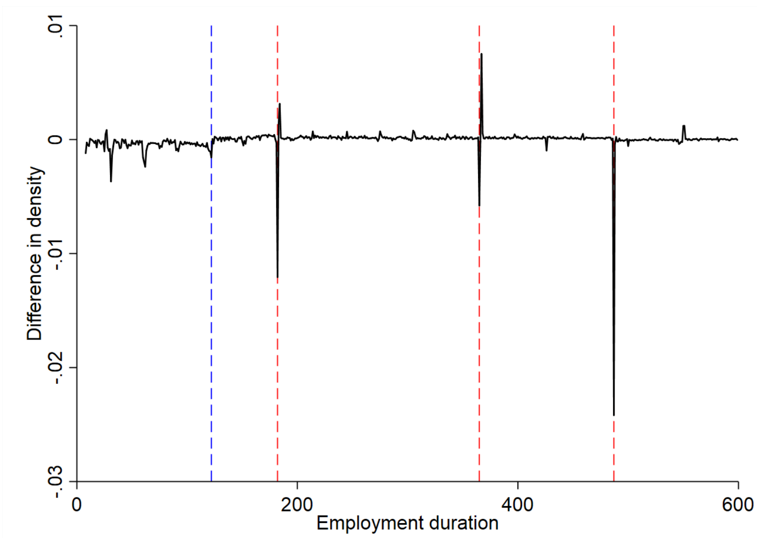


Figure 3: Difference in h density under conventions 2009 and 2006

Note: This graph plots the difference in density between the distribution of employment durations under the conventions 2009 and 2006. The blue dash line corresponds to 4 months, the only discontinuity in the eligibility function under the 2009 convention. The three red dash lines correspond respectively to 6, 12 and 16 months, the three discontinuities in the eligibility function under the 2006 convention.

Table 1 displays some descriptive statistics on workers who enter unemployment with an affiliation around the first thresholds (4 and 6 months). These high risk workers are mainly young employees who are offered fixed-term contracts in sectors which traditionally make intensively use of these unstable contracts.

		4 months		6 months	
		Mean	Sd	Mean	Sd
	Age	25.03	8.01	26.46	8.21
	Daily wage	38.91	27.22	37.32	29.75
		Share		Share	
Sex	Female	48		48	
	Male	52		52	
Contract	Fixed-term	58		48	
	Permanent	14		18	
	Other	26		32	
Class	Worker	35		36	
	Employee	51		43	
	Manager	03		04	
	Other	11		18	
Sector	Admin. serv.	21		22	
	Trade	18		15	
	Hotel Catering	17		14	
	Other	44		49	
	N	24680		34932	

Table 1: Descriptive statistics

Note: The table displays some statistics on workers who enter unemployment with an employment history close to the threshold, 4 or 6 months (112-125, 172-185 days). For the sector, I enumerate only the three most present sectors. The sector in fourth position Construction is clearly less frequently observed (5.56% and 6.09%).

3.2 Employment duration response

A requirement variation modifies the eligibility status of workers located on the threshold, i.e working $h = \theta$ is no longer sufficient. Consequently, those who are not constrained by frictions change their behavior. The model distinguishes three different behavioral changes. First, workers in group 1 recover their natural job duration. Second, workers in group 2 bunch on the new threshold. Third, workers of group 3 are immobilized by frictions. To recover all behavioral changes, I need to identify the level of frictions and the type α which corresponds to group 1, i.e the marginal bunchers. I use a bunching method following Saez (2010) and Kleven and Waseem (2013).

3.2.1 Counterfactual

To recover the distributions of types and to assess the amount of bunching, I need to know what would be the distribution of employment durations in the absence of any discontinuity in the eligibility function. f being the observed distribution of employment durations, I denote f^0 as the counterfactual[7]. The distribution f displayed on figure 10 contains spikes due to the rigidity of employment contracts. Firstly, because contracts are more likely to end at the beginning of the month, a spike is observed on the first of each month. Secondly, high spikes are observed every 6 months due to the duality of the french labor market. It exists two types of contracts: fixed-term contracts and permanent contracts. Every 6 months, a significant share of fixed-term contracts end and can be renewed. At this date, workers hired with a fixed-term contract are more likely to enter unemployment. Thirdly, the behaviors are modified around each discontinuity in the eligibility function.

This counterfactual is approximated by a $\overline{p}^{th}$ degree polynomial after controlling for cyclical separations and for bunching. I introduce a dummy which captures the excess mass of separations on the first day of each month (attached parameter γ_1) and four dummies to control for the separations due to the termination of fixed-term contracts (attached parameters $\gamma_2^{(j)}$ where $j \in \{6, 12, 18, 24\}$). I define the vector χ which contains the number of days corresponding to the first day of each month. To extract the impact of the discontinuity in the eligibility function on the distribution, I control for the effect of the notches on the range noted S. I estimate the following model using an OLS estimator:

$$f_i = \sum_{p=0}^{\overline{p}} \gamma_0^{(p)} (x_i)^p + \gamma_1 \mathbf{1}_{x_i \in \chi} + \sum_{j=\{6,12,18,24\}} \gamma_2^{(j)} \mathbf{1}_{x_i = \chi(j)} + \sum_{l \in S} \gamma_3^{(l)} \mathbf{1}_{x_i = l} + \nu_i$$

where x_i is the employment history of bin i and f_i is the observed density of workers in bin i, i.e whose $h = x_i$. In the case where the range of the notches overlaps a dummy which is attached to the parameter $\gamma_2^{(j)}$, the latter cannot be identified. To recover the missing $\gamma_2^{(j)}$, I use the stability of observed distributions f under different conventions, except at the discontinuities (see figure 3). The missing $\gamma_2^{(j)}$ parameter is recovered by running the same regression on the distribution observed under the other conventions. Thus, it captures the excess mass of separations due to fixed-term contracts and not due to an eligibility discontinuity. Finally, I reconstruct the counterfactual by ignoring the effect of the notche:

$$f_i^0 = \sum_{p=0}^{\overline{p}} \hat{\gamma}_0^{(p)} (x_i)^p + \hat{\gamma}_1 \mathbf{1}_{x_i \in \chi} + \sum_{j=\{6,12,18,24\}} \hat{\gamma}_2^{(j)} \mathbf{1}_{x_i = \chi(j)}$$

The distribution f^0 corresponds to the distribution of types, i.e the distribution of unemployment risks.

[7]Durations are in days.

3.2.2 Identification of the marginal buncher

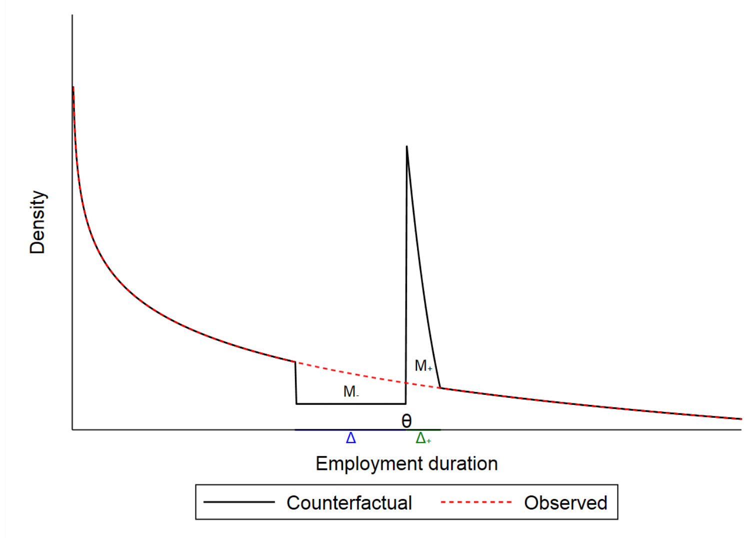


Figure 4: Theoretical distribution in presence of a notche

Note: (Sorry the legend is reversed) This graphic explains theoretically the effect of a notche on the h distribution. The bold black line represents the observed h distribution and the thin red line corresponds to the counterfactual, i.e the shape of a distribution where the discontinuity does not exist. The discontinuity at $h = \theta$ provides incentives to workers naturally located below to choose $h = \theta$. Therefore, workers naturally located between $[\theta - \Delta, \theta[$ bunch on the threshold. The group of workers who naturally choose $h = \theta - \Delta$ are the marginal bunchers, the last workers to bunch. It creates a hole in the distribution represented by the M area. Some workers are stuck due to frictions represented by the ϕ area. Those who bunch on the threshold create an excess mass between θ and $\theta + \Delta_+$, represented by the B area. The mass of workers who are missing must correspond to the excess mass above the threshold: $M = B$.

The first kind of reactions are carried out by the marginal bunchers. They are the lowest risk to bunch because they are indifferent between bunching or remain on their natural location $\theta - \Delta$. In case of a threshold increase, they go back to their natural location. Identifying Δ is key because I firstly estimate their reaction in employment, secondly I obtain some information on the density of my three groups and also on the strength of frictions. The statistic Δ can be approximated using a bunching method (see figure 4 to visualise the notations). Firstly, I compute the excess mass of workers on the threshold by comparing the observed distribution to the counterfactual in the notche range above the threshold (S^+):

$$\hat{M^+} = \sum_{i \in S^+} f_i - f_i^0$$

where $S^+ = [\theta, \theta + \Delta_+]$. The segment where workers bunch Δ_+ is directly observed since the excess mass is very sharp (see figure 5). Then, the missing mass before the notche is defined by:

$$\hat{M^-} = \sum_{i \in S^-} f_i^0 - f_i$$

where $S^- = [\theta - \Delta, \theta[$. Following Chetty et al. (2011), I normalize M^+ and M^- to the average density of the counterfactual distribution over their respective range :

$$\hat{m}^+ = \frac{\hat{M}^+}{\frac{\sum_{i\in S^+} f_i^0}{\Delta_+ + 1}} \quad \hat{m}^- = \frac{\hat{M}^-}{\frac{\sum_{i\in S^-} f_i^0}{\Delta + 1}}$$

Finally, Δ is approximated by iteration until equalizing the missing mass to the excess mass.

3.2.3 Identification of the level of frictions

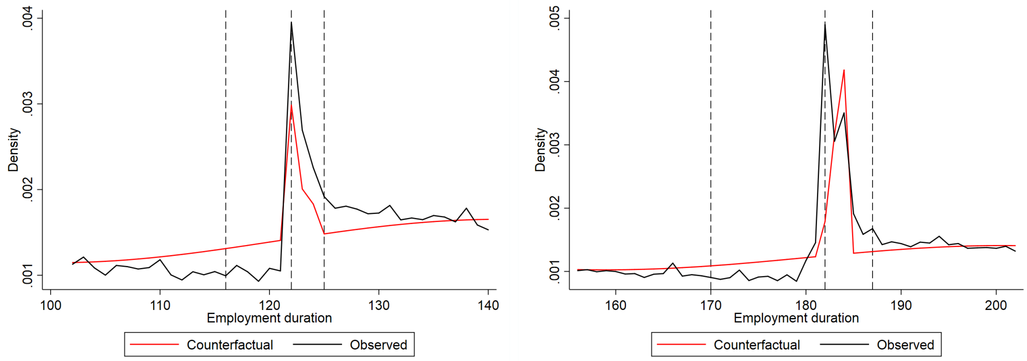
The third kind of reactions are operated by workers who theoretically want to bunch but remain unresponsive due to frictions. For instance, these frictions can be explained by a cognitive bias, an imperfect information or a rigidity in employment contract. Among the pool of workers who want to bunch, these frictions impact randomly a fraction ϕ of them. The level of frictions can be approximated using the missing mass created by the notche. The choices $h \in [\theta - \Delta, \theta[$ are strictly dominated by the choice $h = \theta$. This segment should be completely empty in a frictionless world. Therefore, the remaining workers in the hole are constrained by frictions. I assume that the probability to be unresponsive is constant within the hole, each type α naturally present in the hole are subjected to the same amount of frictions. This assumption is likely to hold since the observed density is flat just before the discontinuity (see figure 5). The probability of being motionless in the hole is defined as:

$$\hat{\phi} = \frac{\int_{\theta-\Delta}^{\theta} f_i \mathrm{d}i}{\int_{\theta-\Delta}^{\theta} f_i^0 \mathrm{d}i}$$

3.2.4 Results

Table 6 displays the results plotted by figure 5. Standards errors are computed using a bootstrap procedure of 500 repetitions. Under the 2006 convention with a $\theta = 182$ days, the marginal bunchers are the workers who work naturally 170 days in their unstable job. Because of the discontinuity created by the minimum employment history requirement, they are indifferent between bunching by working 12 more days to become eligible or working only 170 days and becoming ineligible. In other words, the marginal bunchers are able to increase their job duration by 6.5%. In a frictionless world, all workers whose labor market risk leads them to naturally work between 170 and 181 days bunch on the threshold to get the benefits. Due to the presence of frictions, 84% of them do not manage to fulfill the requirement. Despite the fact that they are better off by bunching, they remain on their natural location. Under the 2009 convention, the propensity to retain an unstable job is lower since the marginal buncher is closer to the threshold (6 days which represents a 4.9% increase). The frictions seem to be also slightly lower around 76%. Two explanations are credible. First, since the

Figure 5: Observed distribution of employment durations and its counterfactual on the notche range



(a) 4 mois

(b) 6 mois

Note: This graphic compares the observed density of h and the counterfactual in the window of the considered notche, i.e 6 months under the 2006 convention. The first dash line corresponds to $h = \theta - \Delta$, i.e it indicates the marginal buncher. The second dash line points out the discontinuity θ. The last dash line indicates the last point of the bunching area $h = \theta + \Delta_{+}$.

	(1)	(2)
Bunching:	4 mois	6 mois
Threshold θ	122	182
Excess mass segment Δ_{+}	3	5
Missing mass segment Δ	6^{***}	12^{***}
	(.4318)	(.6027)
Frictions ϕ	$.76^{***}$	$.84^{***}$
	(.0117)	(.0077)

Standard errors in parentheses (bootstrap 500)

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Figure 6: Bunching results

Note: This table displays for each considered threshold the length of the segment where bunchers located Δ_{+}, the length of the segment where bunchers would be located in absence of the threshold Δ and the level frictions ϕ. $\overline{p} = 5$

threshold is lower under the 2009 convention, the studied population has a higher labor market risk. They are less able to manipulate their affiliation. Second, the reward of exerting an extra effort to catch up with the requirement is lower under the 2009 convention. The PBD is 4 months contrary to 7 months under the 2006 convention. Several robustness checks are performed in appendix B. I vary the specification of the counterfactual (degree, convention) and the excess mass segment. Those checks confirm the obtained results. Furthermore, those results are in line with the literature. Khoury (2019) exploits a jump in the replace rate to 80% at 1 year tenure for one type of contract in France. She shows that the marginal buncheurs increase by 7 to 17% their contract duration. The figures are slightly higher since the population is different, again workers with a high labor market risk are less able to manipulate their affiliation. These results do not argue in favor of a threshold increase. The reactions in terms of employment duration seem limited, increasing the threshold does not provide strong incentives to employees. The benefit of the reform seems low. Frictions are high, increasing the threshold excludes a high share of workers naturally present on the threshold. The cost of the reform seems high.

3.3 Unemployment duration response

Workers who are not able to catch up with the new requirement loose their eligibility and do not receive any unemployment benefit. To identify their reaction in terms of unemployment duration, I exploit the kinks in the replacement rate function following a sharp regression kink design (RKD) as in Card et al. (2012) and Landais (2015).

3.3.1 Strategy

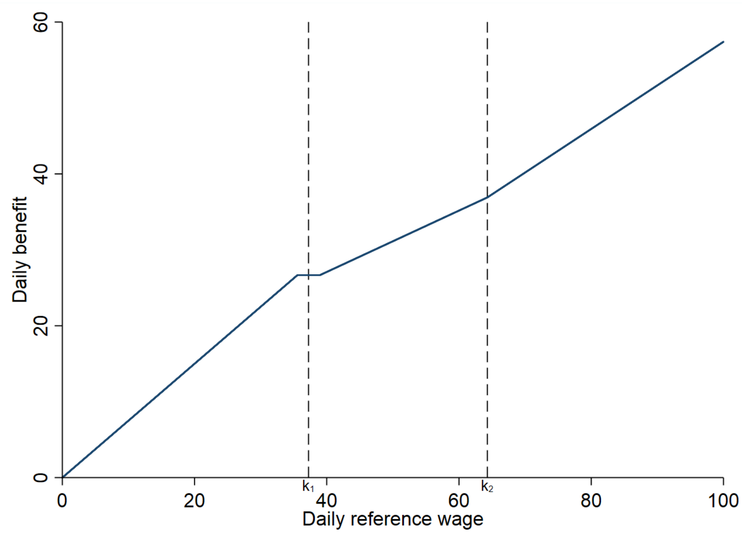


Figure 7: Unemployment benefit pattern in 2009

Note: (sorry it will be transformed in euros 2012) UI rules in France between 2004 and 2012 generate an unemployment benefit which follows the pattern displayed on this graphic. The rule can be summarized by equation 10. The values are expressed in euros 2009, each year the boundaries and the fixed amount C are revalued.

In France, the replacement rate depends on the past daily wage of the new unemployed worker. The daily benefit is either 57.4% of the reference wage or 40.4% plus a fixed amount C. At the end, the replacement rate cannot exceed 75% and the daily benefit should be bounded by a maximum b_{max} and a minimum b_{min}. The relation between the benefit and the reference wage is:

$$b = \min(0.75w, \max(b_{min}, \min(b_{max}, \max(0.404w + C, 0.574w))) \tag{10}$$

Figure 7 shows that the replacement rate function is not smooth when the reference wage varies, therefore some kinks can be exploited to identify the change in behaviors in unemployment due to a change in the unemployment benefit. I focus on one kink: $k_1 = \frac{\frac{b_{min}}{0.75}+\frac{b_{min}-C}{0.404}}{2}$ and use the second kink $k_2 = \frac{C}{0.574-0.404}$ to test the robustness of my results . Since crossing the threshold, i.e becoming eligible, corresponds to increase the unemployment benefit received for a given PBD, I can recover my statistic of interest: ε_{db}.

The idea is to examine the slope of the relationship between the average unemployment duration and the reference wage at the kink by considering the observed kink in the average unemployment duration as the treatment effect of a change in benefit level. The validity of the identification relies on two assumptions. First, the direct marginal effect of the benefit on the unemployment duration should be smooth, i.e it must not have any manipulation of the assignment variable (i.e the reference wage) around the kink. Figure 13 in appendix B shows that the wage distribution is smooth around the kink. As in Landais (2015) and Card et al. (2012), I perform a McCrary test to confirm these observations. On top of that, it seems very unlikely that workers strategically behave and manage to choose a wage in function of the shape of the benefit pattern. In addition to the complexity of the benefit function, workers do not anticipate the unemployment benefit they would get in the event of job loss when negotiating their wage. Second, workers must be similar on both sides of the kink. Therefore, the density of the observed and unobserved heterogeneity should evolve smoothly with the wage at the kink. Since no other changes occur at the kink, only the unemployment duration should change. In appendix B, I show that the age distribution and some other characteristics (sex, types of contract...) are smooth at the kink.

The assignment variable is the reference wage w and the outcome of interest is the unemployment duration d. I denote $\overline{p}$ as the degree of the polynomial and κ as the length of the bandwidth. The regression of the RKD takes the following form:

$$E(d|w) = \mu_0 + \sum_{p=1}^{\overline{p}} \left(\mu_{1,p}(w-k)^p + \mu_{2,p}(w-k)^p \mathbf{1}_{w \geq k} \right) \tag{11}$$

where $|w-k| \geq \kappa$. Coefficient $\mu_{2,1}$ captures the change in slope of the conditional expectation of the unemployment duration at the kink. The local average treatment effect (LATE) is the ratio between this coefficient and the deterministic change in slope of the unemployment benefit observed on figure 7. For the sake of interpretation, I transform the statistic into an elasticity by applying the following transformation: $\varepsilon_{db} = LATE\frac{b(k)}{E(d|k)}$. Restricting my sample to only workers on the threshold, i.e $h = \theta$, does not yield enough power to my estimation. In consequence, I restrict my sample to unemployment spells under the 2004 and 2006 conventions with an employment history

	(1)	(2)	(3)	(4)
	d_1	d_2	d_3	d
LATE	7.44***	26.71***	23.91***	4.19***
	(.939)	(4.794)	(4.851)	(1.061)
ε_b	.946***	1.413***	1.173***	.446***
	(.119)	(.253)	(.238)	(.113)
N	23242	23240	23479	23243

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Unemployment durations response to a benefit change

Note: This table summarizes the results given the regression based on equation 11 for the workers whose h is equal to 6 months between 2004 and 2009. The considered kink is k_1. Each column correspond to a different definition of d: column (1) registered unemployment, column (2) claimed unemployment, column (3) total non-employment and column (4) main definition. $\bar{p} = 1$ and κ=20.

which qualifies them only to a PBD of 7 months ($h \in [182; 365[$). So, I perform this estimation on workers who face the same benefit function and the same PBD. The magnitude of the result is similar to those obtained by the local estimation but the precision is better. Under convention 2009, the benefit pattern changes with the affiliation, so it complicates the estimation.

3.3.2 Results

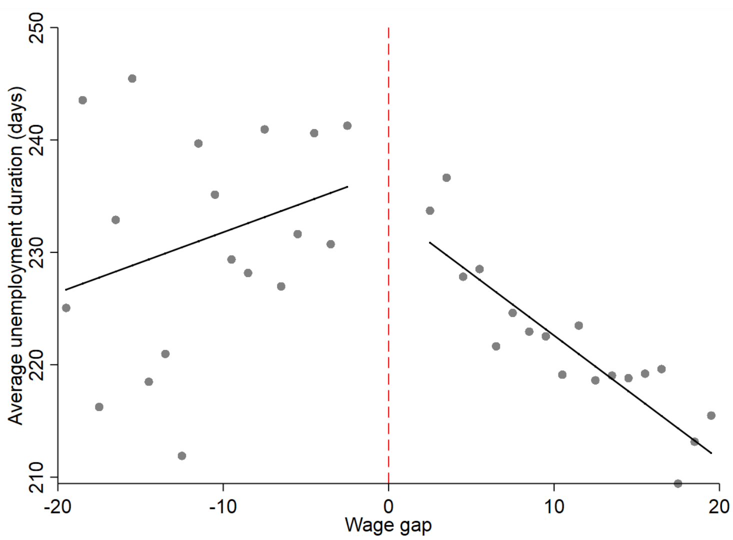


Figure 8: Change in slope at k_1

Note: It exhibits the change in average unemployment duration at kink k_1 estimated by equation 11. Results are available on table 5.

Figure 8 shows that the slope of the relationship between the average unemployment duration and the reference wage changes at kink k_1. The results for the different definition of the unemployment duration are displayed in Table 5. The LATE, i.e the average treatment effect on workers whose employment history qualifies for 7 months and reference wage is close to k_1, indicates that a 1 € increase in the daily unemployment benefit yields a 4 days increase in unemployment duration. In other word, a 1 € increase in the weekly benefit rises the unemployment duration by 0.08 weeks. The elasticity amounts to .45. If the benefit goes up by 10%, the time spent in unemployment rises by 4.5%. This finding is consistent with the results found in literature (see Landais, 2015). Some robustness checks confirm my results in appendix B. I run the same model with different polynomial degree $\overline{p}$ or by changing the bandwidth and on the second kink. Finally, I verify that censoring does not bias my results by running a Tobit model.

4 Optimal empirical eligibility

After having calibrated my formula 8, I assess the welfare implications of a marginal variation in the eligibility threshold. A reform which tightens the eligibility condition has a huge negative effect on high risk workers who are directly impacted and a small positive effect for the rest. I show that, for reasonable assumptions, a threshold increase generates more costs than benefits from the point of view of an utilitarian social planner[8]. The optimal threshold seems to be located below 4 months.

4.1 Calibration

The annual interest rate is assumed to be 5%, $r = 0.013\%$. Each impacted worker enters unemployment with an affiliation equal to the threshold θ. In average, workers on the thresholds remain unemployed for 223 days at 4 months and 320 days at 6 months. The relation between the benefit and the reference wage is unchanged under both conventions. Since the average daily wage is similar within the two populations ($w \approx 48.5$), I just apply the benefit rule (euros 2012): $b = 31.2$. The drop in benefit experienced by impacted workers is defined as the difference between the benefit b and the social minimum income implemented in France over this period: $\frac{\partial b}{\partial \theta} = \frac{15.6-31.2}{31.2} = -0.5$. The tax τ is computed as $\tau = 0.0604w = 3$. Δ, ϕ and $\varepsilon_{d,b}$ are provided by the two last sections. Under convention 2006, $\Delta = 12$ and $\phi = 0.84$. Under convention 2009, $\Delta = 6$ and $\phi = 0.76$. The estimated elasticity of the average unemployment duration with respect to the benefit level can be transform into an elasticity of the job finding rate: $\varepsilon_{d,b} = 0.3 = -\varepsilon_{\lambda,b}$. Thanks to the counterfactual reconstructed in the bunching estimation, I can recover the density of group 1 $a_1 = f^0(\theta-\Delta) - f(\theta-\Delta)$, group 2 $a_2 = \sum_{\alpha=\theta-\Delta+1}^{\theta-1} f^0(\alpha) - f(\alpha)$ and group 3 $a_3 = (1-\phi)f^0(\theta)$. The figures obtained are

[8] The overall effect does not depend on the preference for redistribution of the social planner.

summarized in table 3.

I assume that all workers are hand-to-mouth, they consume their benefit in unemployment and their net wage in employment. It's consistent with the labor market risk that I focus on. Impacted workers have a low ability to self insure and a high probability to loose their unstable jobs so they consume their whole income. Nevertheless, workers who fall into unemployment with a longer affiliation are less likely to adopt a hand-to-mouth behaviour. Their marginal utility in employment is therefore overvalued: their welfare gain of being less taxed is overestimated. In addition, I do not consider the heterogeneity of wages. Since those workers are more likely to earn a high wage, again their marginal utility in employment is overvalued. This overvaluation is not a problem since it goes against my conclusion: despite this overestimation of the benefit of the reform, I conclude that it decreases the overall welfare in the economy. I also assume that the utility function takes the following form: $u(c) = \frac{c^{1-}-1}{1-\sigma}$ where σ corresponds to the risk aversion.

Thanks to those assumptions, in the following, I can compute the consumption smoothing benefit experienced by impacted and unimpacted workers according to different values of risk aversion. I'm also able to calculate the contributions of each group to the tax variation. Thanks to those results, I can figure out how the tax changes when the threshold varies, what is the minimum value of the ratio of the marginal utility in unemployment over the marginal utility in employment which leads to the conclusion of a reduction in well-being through the reform and, finally, how the benefit of the reform is related to its cost for different values of risk aversion.

4.2 Risk-based impact on welfare

i	a_i	CS_i $\sigma = 1$	$\sigma = 2$	$\sigma = 3$	FE_i	FE_i^h	FE_i^λ
				4 months			
1	.024	-3.850	-.124	-.004	$-1.787e^{-3}$	$.001e^{-3}$	$-1.788e^{-3}$
2	.148	-3.850	-.124	-.004	$3.967e^{-3}$	$3.967e^{-3}$	0
3	.824	-3.850	-.124	-.004	$-5.663e^{-3}$	0	$-5.663e^{-3}$
Others	99.004	.026	$.585e^{-3}$	$.013e^{-3}$	0	0	0
				6 months			
1	.14	-4.573	-.147	-.005	$-1.452e^{-3}$	$.002e^{-3}$	$-1.454e^{-3}$
2	.186	-4.573	-.147	-.005	$3.209e^{-3}$	$3.209e^{-3}$	0
3	1.40	-4.573	-.147	-.005	$-7.631e^{-3}$	0	$-7.631e^{-3}$
Others	98.4	.051	.001	$.025e^{-3}$	0	0	0

Table 3: Risk-based welfare analysis

Note: The table displays the value of both sides of equation 9 according to the calibration previously described.

As shown by table 3, there is a double heterogeneity. Firstly, the direct effect of such a reform is heterogeneous within the labor force. First, a majority of workers, around 99%, are not directly impacted by a marginal eligibility change. They do not experience any variation in their compensation but only a change in the tax they pay on their wage. Second, a minority of workers, in particular the highest risk workers, around 1%, are directly impacted by a marginal eligibility change. They are excluded from the system. They experience a variation in their compensation and also a change in the tax they pay when they are employed. By comparing the change in utility in unemployment and the change in utility in employment without taking into account the effect of behavioral responses on the tax, the consumption smoothing loss is painful for excluded workers. The gain in utility of the rest of the population is low relative to the loss experienced by the excluded because the benefit of excluding a minority is shared by the whole population: it represents little individually.

Secondly, the behavioral reactions to the reform is also heterogeneous within the group of impacted workers. Three types of responses exist. The group 1, the marginal bunchers, have too much difficulty catching up with the new requirement. They get back to their natural affiliation and search more intensively for a job. The first behavioral reaction tightens the budget whereas the second increases it. The overall generated fiscal externality is negative: the fiscal externality due to the change in affiliation is lower than the fiscal externality due to the change in job finding rate. The group 2 follows the threshold by increasing his affiliation by the same amount to remain eligible. The effect on the tax is positive since the benefit of increasing the affiliation is surpassed by the cost of receiving higher benefits. Group 2 has a positive fiscal externality. The group 3 is stuck by frictions and tries to find rapidly a job. They contribute to decrease the expenses of the insurance system, the fiscal externality is negative. Since group 3 generates the best externality on the budget, the more fictions there are, the less tax is needed. The more exclusion, the lower the tax. The effect of the change in affiliation on the tax is much lower than the effect of a change in the average unemployment duration. Working more before an unemployment spell generates more taxes (in reality 6.4% of the wage) whereas being less unemployed decreases the expenses by the benefit (around 50% of the wage). Providing the incentive to work more before a job loss is less effective than providing the incentive to remain less unemployed.

The purpose of increasing the eligibility threshold is to provide better incentives, especially to the employees. But manipulating the minimum employment history requirement has one weakness: it provides incentives locally, i.e to a small part of the labor force, in particular to the most risky. Therefore, the cost of this reform is concentrated on the most risky workers and the benefit of their extra efforts are share between everyone. Eligibility puts the responsibility of all opportunistic behaviors on high risk workers. The most fragile workers are used to offset the lack in intensity at work or in job search of the others. Furthermore, the literature (e.g Kolsrud et al. (2018)) shows that high risk workers value more unemployment insurance than others. Since they are less able to self-insure and are less likely to find a job rapidly, they experience a high and durable loss in consumption during their spell in the absence of unemployment insurance. The provision of benefits is essential for them. Therefore, eligibility provides incentives to work more and to search more intensively only to those who value more the public insurance.

4.3 Overall welfare impact

	$\frac{\mathrm{d}\tau}{\mathrm{d}\theta}$	$\frac{u'(c_u)-u'(c_e)}{u'(c_e)}$	$\frac{LHS}{RHS}$ $\sigma = 1$	$\sigma = 2$	$\sigma = 3$
4 months	$-.207e^{-3}$	.254	1.821	4.512	8.436
6 months	$-.425e^{-3}$	.323	1.431	3.570	6.689

Table 4: Overall welfare analysis

Note: The table displays the value of some terms in equation 9 according to the calibration previously described.

By comparing the left hand-side to the right hand-side of equation 9, I can recover the overall welfare impact of a marginal increase in the threshold when it's equal to 4 months or to 6 months. It indicates whether the decrease in well-being experienced by impacted workers is offset by the increase in well-being created by the tax reduction for the whole labor force. In other words, I compare the cost of exclusion to the effects of reactions on the budget constraint.

First of all, increasing the threshold provides effectively better incentives since the tax goes down. Since the reactions in employment are stronger at 6 months, and more workers are located on the threshold, the tax variation is higher at 6 months than at 4 months. A 1% increase in the threshold decreases the tax by 0.02% at 4 months and by 0.04% at 6 months. Then, from the social planner's point of view, the loss in consumption smoothing capacity for a minority of workers is more painful than the gain of cutting the tax for all workers. The cost-benefit ratio of the reform increases with the value of risk aversion. For a logarithmic utility function, the cost represents 80% of the benefit at 4 months and 43% at 6 months. The gap between the cost and the benefit is higher when the threshold is at 6 months. Increasing the threshold at 6 months is worse than increasing the threshold at 4 months. Finally, I compute the ratio of marginal utilities which equalizes the gains to the losses. Increasing the threshold rises the total well being if this ratio is above 25% at 4 months and 32% at 6 months. In other words, if the ratio of the difference in marginal utilities over the marginal utility in employment of impacted workers is above the bound, the reform is welfare decreasing. Again, the welfare increase experienced by low risk workers is overvalued and impacted workers are likely to be hand to mouth. The true bound on the ratio may be lower. Therefore, the optimal threshold is below 4 months if the ratio of marginal utilities is higher than 32%. The optimal threshold is located between 4 and 6 months if the ratio lies within 25 and 32%. The optimal threshold is higher than 6 months if the ratio is below 25%.

5 Conclusion

The implementation of a minimum employment history criterion is based on the existence of strategic behaviors in employment. Nevertheless, I show that affiliations react little to the incentives given by the public insurance. Hence, the costs of implementing such a criterion encompass the benefits: this policy instrument is inefficient. The exclusion of workers who are constrained by frictions or by their unemployment risk has more negative consequences than the benefits of the tax cut resulting from better incentives given to impacted employees and excluded workers. Decreasing the threshold below 4 months would enhance the total welfare in the economy. The welfare rise would be even higher if the social planner exhibits a stronger taste for redistribution since the well being of high risk workers would be more weighted.

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Appendices

A Model algebra

A.1 Discrete Time

The value of being unemployed in discrete time is:

$$\begin{aligned} U(A_0,h) =& u(\underline{c}_0) + \sum_{t=1}^{\infty} \beta^t (1-\lambda)^t u(\underline{c}_t) + \sum_{t=1}^{\infty} \beta^t (1-\lambda)^{(t-1)} \lambda u(rA_t + w - \tau) \\ U(A_0,h) =& u(\underline{c}_0)dt + \sum_{t=1}^{\infty} \left(\frac{1-\lambda dt}{1+rdt}\right)^t u(\underline{c}_t)dt + \frac{\lambda dt}{(1-\lambda dt)} \sum_{t=1}^{\infty} \left(\frac{1-\lambda dt}{1+rdt}\right)^t u(rA_t + w - \tau) \end{aligned}$$

By taking the limit $\lim_{dt \to 0} U(A_0,h)$, I get the value function in continuous time.

A.2 Definition of the b function

I approximate the true benefit pattern with a continuous function of the form:

$$\begin{aligned} b(h;\theta) &= \mathbf{1}[h < \theta]\underline{b} + \mathbf{1}[h \geq \theta]b \approx \underline{b} + P(h;\theta)(b - \underline{b}) \\ \text{where } P(h;\theta) &= \left(\frac{1}{2} + \frac{1 - e^{-100(h-\theta)}}{2(1 + e^{-100(h-\theta)})}\right) \end{aligned}$$

Notice that: $\frac{\partial b(h;\theta)}{\partial h}|_{h=\theta} = -\frac{\partial b(h;\theta)}{\partial \theta}|_{h=\theta}$ and $\underline{b}$ corresponds to the social minimum income. I assume that $\frac{\partial b(h;\theta)}{\partial h}|_{h \neq \theta} \approx 0$ and $\frac{\partial b(h;\theta)}{\partial \theta}|_{h \neq \theta} \approx 0$.

A.3 Value of a stable job

The present value of being employed for eternity is:

$$\begin{aligned} &\max_{\bar{c}_s} \int_0^{\infty} e^{-rs} u(\bar{c}_s) \mathrm{d}s \\ s.t\ &A_0 + \int_0^{\infty} e^{-rs} (w - \tau - \overline{c_s}) \mathrm{d}s \geq 0 \\ &\lim_{s \to \infty} e^{-rs} A_s \geq 0 \\ &A_s = e^{rs} \left(A_0 + \int_0^s e^{-rt} (w - \tau - \overline{c_t}) dt\right) \end{aligned}$$

The first order condition yields: $u'(\bar{c}_s) = \mu_{BC}$. Therefore, $\bar{c}$ is constant overtime. From the budget constraint I obtain $\bar{c} = rA_0 + w - \tau$. The value function can be rewritten as $\frac{u(rA_0 + w - \tau)}{r}$.

A.4 Optimal choices

After plugging $U_\alpha(A_h, h)$ in $V_\alpha(A)$ to obtain the present value of the lifetime consumption of a given worker:

$$
\begin{aligned}
V_\alpha(A_0) = \max_{\underline{c_t}, c_t, h, \lambda} \int_0^h e^{-rs} u(\underline{c_s}) \mathrm{d}s + e^{-rh} \left(\int_0^\infty u(c_t) e^{-(r+\lambda)t} dt + \lambda \int_0^\infty \frac{u(rA_x + w - \tau)}{r} e^{-(r+\lambda)x} dx - \psi_\alpha^u(\lambda) \right) - \psi_\alpha^e(h) \\
\text{s.t } A_0 + \int_0^h e^{-rs}(w - \tau - \underline{c_s}) \mathrm{d}s + e^{-rh} \int_0^\infty e^{-rt}(b_t(h;\theta) - c_t) dt \geq 0 \\
\lim_{s\to\infty} A_s e^{-rs} \geq 0 \\
\text{where } A_t = e^{r(t+h)} \left(A_0 + \int_0^h (w - \tau - \underline{c}_s) e^{-rs} \mathrm{d}s + e^{-rh} \int_0^t (b_x(h;\theta_0) - c_x) e^{-rx} dx \right)
\end{aligned}
$$

By defining a Lagrangian multiplier for each constraint, e.g μ_{BC} for the budget constraint and μ_{CC} for the transitivity condition, I obtain equations 1 and 2.

A.5 Optimal θ

The social planner maximizes the lifetime discounted utility of workers which composed its economy under the constraint of a balanced budget. I define R_b^α as the discounted unemployment duration of an individual whose labor market risk is α:

$$
R_b^\alpha = \frac{e^{-rh_\alpha}}{r + \lambda_\alpha}
$$

I define R_w^α as the sum of discounted employment periods of an individual whose labor market risk is α:

$$
R_w^\alpha = \frac{1}{r} - \frac{e^{-rh_\alpha}}{r + \lambda_\alpha}
$$

A.5.1 Homogeneous labor market risk without frictions

Assume no frictions and that it exists only one labor market risk α. The social planner's program is:

$$
\begin{aligned}
&\max_{\theta,\tau} V_\alpha(A_0; \theta, \tau) \\
&s.t\ \tau R_w^\alpha = b(h_\alpha; \theta) R_b^\alpha
\end{aligned}
$$

The first order condition yields:

$$
\begin{aligned}
\frac{\mathrm{d}V_\alpha}{\mathrm{d}\theta} &= 0 \;\rightarrow\; \frac{\partial V_\alpha}{\partial\theta} + \frac{\partial V_\alpha}{\partial\tau}\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = 0 \\
\text{where } \frac{\partial V_\alpha}{\partial\theta} &= e^{-rh_\alpha}\int_0^\infty e^{-(r+\lambda_\alpha)}u'(c_t)\mathrm{d}t\frac{\partial b(h_\alpha;\theta)}{\partial\theta} \\
\frac{\partial V_\alpha}{\partial\tau} &= -\left(u'(\underline{c}_\alpha)\left(\frac{1-e^{-rh_\alpha}}{r}\right) + e^{-rh_\alpha}\lambda_\alpha\int_0^\infty e^{-(r+\lambda_\alpha)t}\frac{u'(\bar{c}_{t,\alpha})}{r}\mathrm{d}t\right)
\end{aligned}
$$

The total effect of the tax change is:

$$
\begin{aligned}
\frac{\mathrm{d}\tau}{\mathrm{d}\theta} &= \frac{\partial\tau}{\partial\theta} + \frac{\partial h_\alpha}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}h_\alpha} + \frac{\partial\lambda_\alpha}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_\alpha} \\
\text{where } \frac{\partial\tau}{\partial\theta} &= \frac{\partial b(h_\alpha;\theta)}{\partial\theta}\frac{R_b^\alpha}{R_w^\alpha} \\
\frac{\mathrm{d}\tau}{\mathrm{d}h_\alpha} &= -b(h_\alpha;\theta)\frac{R_b^\alpha}{{R_w^\alpha}^2} + \frac{\partial b(h_\alpha;\theta)}{\partial h_\alpha}\frac{R_b^\alpha}{R_w^\alpha} \\
\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_\alpha} &= -\frac{b(h_\alpha;\theta)}{r(r+\lambda_\alpha)}\frac{R_b^\alpha}{{R_w^\alpha}^2}
\end{aligned}
$$

The optimal θ is characterized by the following trade-off:

$$
\begin{aligned}
&\frac{\partial V_\alpha}{\partial\theta} + \frac{\partial V_\alpha}{\partial\tau}\frac{\partial\tau}{\partial\theta} = \left(-\frac{\partial V_\alpha}{\partial\tau}\right)\left(\frac{\partial h_\alpha}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}h_\alpha} + \frac{\partial\lambda_\alpha}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_\alpha}\right) \\
\rightarrow &e^{-rh_\alpha}\frac{\partial b(h_\alpha;\theta)}{\partial\theta}\left(\int_0^\infty e^{-(r+\lambda_\alpha)t}u'(c_{t,\alpha})\mathrm{d}t - \frac{1}{(r+\lambda_\alpha)}\left(\frac{u'(\underline{c}_\alpha)\left(\frac{1-e^{-rh_\alpha}}{r}\right) + e^{-rh_\alpha}\lambda_\alpha\int_0^\infty e^{-(r+\lambda_\alpha)t}\frac{u'(\bar{c}_{t,\alpha})}{r}\mathrm{d}t}{R_w^\alpha}\right)\right) \\
= &\left[\frac{\partial h_\alpha}{\partial\theta}\left(-b(h_\alpha;\theta)\frac{R_b^\alpha}{{R_w^\alpha}^2} + \frac{\partial b(h_\alpha;\theta)}{\partial h_\alpha}\frac{R_b^\alpha}{R_w^\alpha}\right) + \frac{\partial\lambda_\alpha}{\partial\theta}\left(-\frac{b(h_\alpha;\theta)}{r(r+\lambda_\alpha)}\frac{R_b^\alpha}{{R_w^\alpha}^2}\right)\right] \\
&\left(u'(\underline{c}_\alpha)\left(\frac{1-e^{-rh_\alpha}}{r}\right) + e^{-rh_\alpha}\lambda_\alpha\int_0^\infty e^{-(r+\lambda_\alpha)t}\frac{u'(\bar{c}_{t,\alpha})}{r}\mathrm{d}t\right) \\
\rightarrow &CS_\alpha = -\frac{\partial V_\alpha}{\partial\tau}FE_\alpha
\end{aligned}
$$

The consumption smoothing loss is defined as:

$$
CS_\alpha = e^{-rh_\alpha}\frac{\partial b(h_\alpha;\theta)}{\partial\theta}\left(\int_0^\infty e^{-(r+\lambda_\alpha)t}u'(c_{t,\alpha})\mathrm{d}t - \frac{1}{(r+\lambda_\alpha)}\left(\frac{u'(\underline{c}_\alpha)\left(\frac{1-e^{-rh_\alpha}}{r}\right) + e^{-rh_\alpha}\lambda_\alpha\int_0^\infty e^{-(r+\lambda_\alpha)t}\frac{u'(\bar{c}_{t,\alpha})}{r}\mathrm{d}t}{R_w^\alpha}\right)\right)
$$

. The fiscal externality is defined as:

$$
FE_\alpha = \frac{\partial h_\alpha}{\partial\theta}\left(-b(h_\alpha;\theta)\frac{R_b^\alpha}{{R_w^\alpha}^2} + \frac{\partial b(h_\alpha;\theta)}{\partial h_\alpha}\frac{R_b^\alpha}{R_w^\alpha}\right) + \frac{\partial\lambda_\alpha}{\partial\theta}\left(-\frac{b(h_\alpha;\theta)}{r(r+\lambda_\alpha)}\frac{R_b^\alpha}{{R_w^\alpha}^2}\right)
$$

. So optimal θ maximizes the duration in employment of workers under the constraint that they remain compensated. In the case where the consumption of workers α can be approximated by a hand to mouth consumption (note $c_u = b$ and $c_e = w - \tau$), the optimal θ becomes:

$$\frac{\partial b(h_\alpha;\theta)}{\partial \theta}\frac{u'(c_u)-u'(c_e)}{u'(c_e)} = -\left(\frac{\partial \lambda_\alpha}{\partial \theta}\frac{1}{r(r+\lambda_\alpha)} + \frac{\partial h_\alpha}{\partial \theta}\right)\frac{b(h_\alpha;\theta)}{R_w^\alpha} + \frac{\partial b(h_\alpha;\theta)}{\partial h_\alpha}\frac{\partial h_\alpha}{\partial \theta}$$

A.5.2 Heterogeneous labor market risk without frictions

Assume no frictions and an heterogeneity in the labor market risk faced by each agent. The program of the social planner is:

$$\begin{aligned}
&\max_{\theta,\tau} \int_0^\infty V_\alpha(A_0;\theta,\tau)\mathrm{d}\mathcal{A}(\alpha)\\
&s.t\ \tau\int_0^\infty R_w^\alpha \mathrm{d}\mathcal{A}(\alpha) = \int_0^\infty b(h_\alpha;\theta)R_b^\alpha \mathrm{d}\mathcal{A}(\alpha)
\end{aligned}$$

The first order condition yields:

$$\begin{aligned}
&\frac{\partial \int_0^\infty V_\alpha \mathrm{d}\mathcal{A}(\alpha)}{\partial\theta} + \frac{\int_0^\infty \partial V_\alpha \mathrm{d}\mathcal{A}(\alpha)}{\partial \tau}\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = 0\\
\text{where }&\frac{\partial \int_0^\infty V_\alpha \mathrm{d}\mathcal{A}(\alpha)}{\partial\theta} = \int_0^\infty \frac{\partial V_\alpha}{\partial\theta}\mathrm{d}\mathcal{A}(\alpha) = \int_{\theta-\Delta}^\theta \frac{\partial V_\alpha}{\partial \theta}\mathrm{d}\mathcal{A}(\alpha)\\
&\frac{\partial \int_0^\infty V_\alpha \mathrm{d}\mathcal{A}(\alpha)}{\partial\tau} = \int_0^\infty \frac{\partial V_\alpha}{\partial\tau}\mathrm{d}\mathcal{A}(\alpha) = \int_0^\infty \frac{\partial V_\alpha}{\partial \tau}\mathrm{d}\mathcal{A}(\alpha)\\
\text{so }&\int_{\theta-\Delta}^\theta e^{-rh_\alpha}\int_0^\infty e^{-(r+\lambda_\alpha)}u'(c_{t,\alpha})\mathrm{d}t\frac{\partial b(h_\alpha;\theta)}{\partial\theta}\mathrm{d}\mathcal{A}(\alpha)\\
=&\frac{\mathrm{d}\tau}{\mathrm{d}\theta}\int_0^\infty\left(u'(\underline{c}_\alpha)\left(\frac{1-e^{-rh_\alpha}}{r}\right) + e^{-rh_\alpha}\lambda_\alpha\int_0^\infty e^{-(r+\lambda_\alpha)t}\frac{u'(\overline{c}_{t,\alpha})}{r}\mathrm{d}t\right)\mathrm{d}\mathcal{A}(\alpha)
\end{aligned}$$

The left hand-side describes the variation in benefits without any behavioral response and the right hand-side reports the total effect of the tax change (mechanical plus behavioral effects) on the employment periods. The total effect of a threshold change on the tax is:

$$\begin{aligned}
&\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = \frac{\partial\tau}{\partial\theta} + \int_{\theta-\Delta}^\theta\left(\frac{\partial h_\alpha}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}h_\alpha} + \frac{\partial\lambda_\alpha}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_\alpha}\right)\mathrm{d}\mathcal{A}(\alpha)\\
\text{where }&\frac{\partial\tau}{\partial\theta} = \frac{\int_{\theta-\Delta}^\theta \frac{\partial b(h_x;\theta)}{\partial\theta}R_b^x \mathrm{d}\mathcal{A}(x)}{\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x)}\\
&\frac{\mathrm{d}\tau}{\mathrm{d}h_\alpha} = -rR_b^\alpha\left(\frac{b(h_\alpha;\theta)}{\int_0^\infty R_w^x\mathrm{d}\mathcal{A}(x)} + \frac{\int_0^\infty b(h_x;\theta)R_b^x\mathrm{d}\mathcal{A}(x)}{(\int_0^\infty R_w^x\mathrm{d}\mathcal{A}(x))^2}\right) + \frac{\partial b(h_\alpha;\theta)}{\partial h_\alpha}\frac{R_b^\alpha}{\int_0^\infty R_w^x\mathrm{d}\mathcal{A}(x)}\\
&\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_\alpha} = -\frac{R_b^\alpha}{(r+\lambda_\alpha)}\left(\frac{b(h_\alpha;\theta)}{\int_0^\infty R_w^x\mathrm{d}\mathcal{A}(x)} + \frac{\int_0^\infty b(h_x;\theta)R_b^x\mathrm{d}\mathcal{A}(x)}{(\int_0^\infty R_w^x\mathrm{d}\mathcal{A}(x))^2}\right)
\end{aligned}$$

Since only workers with an affiliation equal to the threshold are impacted and want to react, the tax variation is composed by three components: the mechanical effect, the effect of a change in affiliation and the effect of a change in job finding rate. I define ω

The optimal θ is characterized by the following trade-off:

$$\int_{\theta-\Delta}^{\theta} \frac{\partial V_\alpha}{\partial \theta} \mathrm{d}\mathcal{A}(\alpha) + \int_0^\infty \frac{\partial V_\alpha}{\partial \tau}\frac{\partial \tau}{\partial \theta} \mathrm{d}\mathcal{A}(\alpha) = \int_0^\infty \left(-\frac{\partial V_\alpha}{\partial \tau}\right) \mathrm{d}\mathcal{A}(\alpha) \int_{\theta-\Delta}^{\theta} \left(\frac{\partial h_\alpha}{\partial \theta}\frac{\mathrm{d}\tau}{\mathrm{d}h_\alpha} + \frac{\partial \lambda_\alpha}{\partial \theta}\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_\alpha}\right) \mathrm{d}\mathcal{A}(\alpha)$$

$$\begin{aligned}
\rightarrow &\int_{\theta-\Delta}^{\theta} \frac{\partial V_\alpha}{\partial \theta} + \frac{\partial V_\alpha}{\partial \tau}\frac{\partial \tau}{\partial \theta} \mathrm{d}\mathcal{A}(\alpha) + \int_{\alpha \notin [\theta-\Delta;\Delta]} \frac{\partial V_\alpha}{\partial \tau}\frac{\partial \tau}{\partial \theta} \mathrm{d}\mathcal{A}(\alpha) \\
&= \int_0^\infty \left(-\frac{\partial V_\alpha}{\partial \tau}\right) \mathrm{d}\mathcal{A}(\alpha) \int_{\theta-\Delta}^{\theta} \left(\frac{\partial h_\alpha}{\partial \theta}\frac{\mathrm{d}\tau}{\mathrm{d}h_\alpha} + \frac{\partial \lambda_\alpha}{\partial \theta}\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_\alpha}\right) \mathrm{d}\mathcal{A}(\alpha)
\end{aligned}$$

$$\begin{aligned}
\rightarrow &\int_{\theta-\Delta}^{\theta} \Bigg[e^{-rh_\alpha} \frac{\partial b(h_\alpha;\theta)}{\partial \theta} \int_0^\infty e^{-(r+\lambda_\alpha)t} u'(c_{t,\alpha}) \mathrm{d}t \\
&- \left(u'(\underline{c}_\alpha)\left(\frac{1-e^{-rh_\alpha}}{r}\right) + e^{-rh_\alpha}\lambda_\alpha \int_0^\infty e^{-(r+\lambda_\alpha)t}\frac{u'(\overline{c}_{t,\alpha})}{r}\mathrm{d}t\right)\left(\frac{\int_{\theta-\Delta}^{\theta} \frac{\partial b(h_x;\theta)}{\partial \theta} R_b^x \mathrm{d}\mathcal{A}(x)}{\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x)}\right)\Bigg] \mathrm{d}\mathcal{A}(\alpha) \\
&- \int_{\alpha \notin [\theta-\Delta;\theta]} \left(u'(\underline{c}_\alpha)\left(\frac{1-e^{-rh_\alpha}}{r}\right) + e^{-rh_\alpha}\lambda_\alpha \int_0^\infty e^{-(r+\lambda_\alpha)t}\frac{u'(\overline{c}_{t,\alpha})}{r}\mathrm{d}t\right) \mathrm{d}\mathcal{A}(\alpha) \left(\frac{\int_{\theta-\Delta}^{\theta} \frac{\partial b(h_x;\theta)}{\partial \theta} R_b^x \mathrm{d}\mathcal{A}(x)}{\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x)}\right) \\
= &\int_{\theta-\Delta}^{\theta} \Bigg[\frac{\partial h_\alpha}{\partial \theta}\left(-rR_b^\alpha\left(\frac{b(h_\alpha;\theta)}{\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x)} + \frac{\int_0^\infty b(h_x;\theta) R_b^x \mathrm{d}\mathcal{A}(x)}{(\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x))^2}\right) + \frac{\partial b(h_\alpha;\theta)}{\partial h_\alpha}\frac{R_b^\alpha}{\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x)}\right) \\
&+ \frac{\partial \lambda_\alpha}{\partial \theta}\left(-\frac{R_b^\alpha}{(r+\lambda_\alpha)}\left(\frac{b(h_\alpha;\theta)}{\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x)} + \frac{\int_0^\infty b(h_x;\theta) R_b^x \mathrm{d}\mathcal{A}(x)}{(\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x))^2}\right)\right)\Bigg] \mathrm{d}\mathcal{A}(\alpha) \\
&\int_0^\infty \left(u'(\underline{c}_\alpha)\left(\frac{1-e^{-rh_\alpha}}{r}\right) + e^{-rh_\alpha}\lambda_\alpha \int_0^\infty e^{-(r+\lambda_\alpha)t}\frac{u'(\overline{c}_{t,\alpha})}{r}\mathrm{d}t\right) \mathrm{d}\mathcal{A}(\alpha)
\end{aligned}$$

$$\rightarrow a_1 CS_1 + a_2 CS_2 + CS_{others} = -\int_0^\infty \frac{\partial V_\alpha}{\partial \tau} \mathrm{d}\mathcal{A}(\alpha)(a_1 FE_1 + a_2 FE_2)$$

The consumption smoothing loss of workers in the group $i \in \{1,2\}$ is:

$$\begin{aligned}
CS_i = &e^{-rh_i} \frac{\partial b(h_i;\theta)}{\partial \theta} \int_0^\infty e^{-(r+\lambda_i)t} u'(c_{t,i}) \mathrm{d}t \\
&- \left(u'(\underline{c}_i)\left(\frac{1-e^{-rh_i}}{r}\right) + e^{-rh_i}\lambda_i \int_0^\infty e^{-(r+\lambda_i)t}\frac{u'(\overline{c}_{t,i})}{r}\mathrm{d}t\right)\left(\frac{\int_{\theta-\Delta}^{\theta} \frac{\partial b(h_x;\theta)}{\partial \theta} R_b^x \mathrm{d}\mathcal{A}(x)}{\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x)}\right)
\end{aligned}$$

The consumption smoothing gain of unimpacted workers is:

$$CS_{others} = -\int_{\alpha \notin [\theta-\Delta;\theta]} \left(u'(\underline{c}_\alpha) \left(\frac{1-e^{-rh_\alpha}}{r} \right) + e^{-rh_\alpha} \lambda_\alpha \int_0^\infty e^{-(r+\lambda_\alpha)t} \frac{u'(\bar{c}_{t,\alpha})}{r} \mathrm{d}t \right) \mathrm{d}\mathcal{A}(\alpha) \left(\frac{\int_{\theta-\Delta}^\theta \frac{\partial b(h_x;\theta)}{\partial \theta} R_b^x \mathrm{d}\mathcal{A}(x)}{\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x)} \right)$$

The fiscal externality generated by each group i of impacted workers is:

$$FE_i = \frac{\partial h_i}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}h_i} + \frac{\partial \lambda_i}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}\lambda_i}$$

Despite appearances, the trade-off is easily readable. The left hand-side describes the change in utility at work and in unemployment without taking into account the behavioral changes. It's composed by the change in compensation experienced by the impacted workers and the resulting lower tax effect on each employee. The right hand-side takes into account the reactions. Those reactions are generated by the impacted workers and the benefit of their extra effort are distributed over the labor force. If the cost of exclusion encompasses the benefit of mechanically lower the tax plus the benefit of new behaviors, increasing the threshold is welfare reducing.

By assuming that every worker has a hand to mouth behavior (note $c_u = b$ and $c_e = w - \tau$) and by normalizing by $u'(c_e)$ the formula can be simplified as:

$$\begin{aligned}
&\int_{\theta-\Delta}^\theta \frac{\partial b(h_\alpha;\theta)}{\partial \theta} R_b^\alpha \left(\frac{u'(c_u) - u'(c_e)}{u'(c_e)} \right) \mathrm{d}\mathcal{A}(\alpha) \\
= &\int_{\theta-\Delta}^\theta \Bigg[\frac{\partial h_\alpha}{\partial \theta} \left(-r R_b^\alpha \left(b(h_\alpha;\theta) + \frac{\int_0^\infty b(h_x;\theta) R_b^x \mathrm{d}\mathcal{A}(x)}{\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x)} \right) + \frac{\partial b(h_\alpha;\theta)}{\partial h_\alpha} R_b^\alpha \right) \\
&+ \frac{\partial \lambda_\alpha}{\partial \theta} \left(-\frac{R_b^\alpha}{(r+\lambda_\alpha)} \left(b(h_\alpha;\theta) + \frac{\int_0^\infty b(h_x;\theta) R_b^x \mathrm{d}\mathcal{A}(x)}{\int_0^\infty R_w^x \mathrm{d}\mathcal{A}(x)} \right) \right) \Bigg] \mathrm{d}\mathcal{A}(\alpha)
\end{aligned}$$

A.5.3 Heterogeneous labor market risk with frictions

The program of the social planner is:

$$\begin{aligned}
&\max_{\theta,\tau} W(\theta,\tau) = \int_0^\infty \phi \tilde{V}_\alpha(A_0;\theta,\tau) + (1-\phi) V_\alpha(A_0;\theta,\tau) \mathrm{d}\mathcal{A}(\alpha) \\
&s.t\ \tau \int_0^\infty \phi \tilde{R}_w^\alpha + (1-\phi) R_w^\alpha \mathrm{d}\mathcal{A}(\alpha) = \int_0^\infty \phi b(\alpha;\theta) \tilde{R}_b^\alpha + (1-\phi) b(h_\alpha;\theta) R_b^\alpha \mathrm{d}\mathcal{A}(\alpha)
\end{aligned}$$

where $\tilde{R}_w^\alpha = \frac{e^{-r\alpha}}{r+\tilde{\lambda}}$, $\tilde{R}_b^\alpha = \frac{1}{r} - \frac{e^{-r\alpha}}{r+\tilde{\lambda}}$ and $\tilde{V}(A_0)$ is the present discounted value of the lifetime utility of a constrained worker such that

$$\tilde{V}_\alpha(A_0) = \int_0^\alpha e^{-rs} u(\underline{\tilde{c}_s^\alpha}) \mathrm{d}s + e^{-r\alpha} \left(\int_0^\infty u(\tilde{c}_t) e^{-(r+\tilde{\lambda})t} dt + \tilde{\lambda} \int_0^\infty \frac{u(rA_x + w - \tau)}{r} e^{-(r+\tilde{\lambda})x} dx - \psi_\alpha^u(\tilde{\lambda}) \right) - \psi_\alpha^e(\alpha)$$

. The first order condition yields:

$$\frac{\partial \int_0^\infty \phi\tilde{V}_\alpha + (1-\phi)V_\alpha \mathrm{d}\mathcal{A}(\alpha)}{\partial\theta} + \frac{\partial \int_0^\infty \phi\tilde{V}_\alpha + (1-\phi)V_\alpha \mathrm{d}\mathcal{A}(\alpha)}{\partial\tau}\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = 0$$

$$\rightarrow \int_{\theta-\Delta}^{\theta} \phi\left(e^{-r\alpha}\int_0^\infty e^{-(r+\tilde{\lambda}_\alpha)}u'(\tilde{c}_{t,\alpha})\mathrm{d}t\frac{\partial b(\alpha;\theta)}{\partial\theta}\right) + (1-\phi)\left(e^{-rh_\alpha}\int_0^\infty e^{-(r+\lambda_\alpha)}u'(c_{t,\alpha})\mathrm{d}t\frac{\partial b(h_\alpha;\theta)}{\partial\theta}\right)\mathrm{d}\mathcal{A}(\alpha)$$
$$= \frac{\mathrm{d}\tau}{\mathrm{d}\theta}\int_0^\infty \phi\left(u'(\underline{\tilde{c}}_\alpha)\left(\frac{1-e^{-r\alpha}}{r}\right) + e^{-r\alpha}\tilde{\lambda}_\alpha\int_0^\infty e^{-(r+\tilde{\lambda}_\alpha)t}\frac{u'(\bar{\tilde{c}}_{t,\alpha})}{r}\mathrm{d}t\right)$$
$$+ (1-\phi)\left(u'(\underline{c}_\alpha)\left(\frac{1-e^{-rh_\alpha}}{r}\right) + e^{-rh_\alpha}\lambda_\alpha\int_0^\infty e^{-(r+\lambda_\alpha)t}\frac{u'(\bar{c}_{t,\alpha})}{r}\mathrm{d}t\right)\mathrm{d}\mathcal{A}(\alpha)$$

The total effect of a threshold change on the tax is:

$$\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = \frac{\partial\tau}{\partial\theta} + \phi\int_{\theta-\Delta}^{\theta}\left(\frac{\partial\tilde{\lambda}_\alpha}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}\tilde{\lambda}_\alpha}\right)\mathrm{d}\mathcal{A}(\alpha) + (1-\phi)\int_{\theta-\Delta}^{\theta}\left(\frac{\partial h_\alpha}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}h_\alpha} + \frac{\partial\lambda_\alpha}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_\alpha}\right)\mathrm{d}\mathcal{A}(\alpha)$$

where $$\frac{\partial\tau}{\partial\theta} = \frac{\int_{\theta-\Delta}^{\theta}(1-\phi)\frac{\partial b(h_x;\theta)}{\partial\theta}R_b^x\mathrm{d}\mathcal{A}(x) + \phi a(\theta)\frac{\partial b(\theta;\theta)}{\partial\theta}\tilde{R}_b^\theta}{\int_0^\infty \phi\tilde{R}_w^x + (1-\phi)R_w^x\mathrm{d}\mathcal{A}(x)}$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}h_\alpha} = -(1-\phi)rR_b^\alpha\left(\frac{b(h_\alpha;\theta)}{\int_0^\infty \phi\tilde{R}_w^x + (1-\phi)R_w^x\mathrm{d}\mathcal{A}(x)} + \frac{\int_0^\infty \phi b(x;\theta)\tilde{R}_w^x + (1-\phi)b(h_x;\theta)R_b^x\mathrm{d}\mathcal{A}(x)}{(\int_0^\infty \phi\tilde{R}_w^x + (1-\phi)R_w^x\mathrm{d}\mathcal{A}(x))^2}\right)$$
$$+ (1-\phi)\frac{\partial b(h_\alpha;\theta)}{\partial h_\alpha}\frac{R_b^\alpha}{\int_0^\infty \phi\tilde{R}_w^x + (1-\phi)R_w^x\mathrm{d}\mathcal{A}(x)}$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_\alpha} = -(1-\phi)\frac{R_b^\alpha}{(r+\lambda_\alpha)}\left(\frac{b(h_\alpha;\theta)}{\int_0^\infty \phi\tilde{R}_w^x + (1-\phi)R_w^x\mathrm{d}\mathcal{A}(\alpha)} + \frac{\int_0^\infty \phi b(x;\theta)\tilde{R}_b^x + (1-\phi)b(h_x;\theta)R_b^x\mathrm{d}\mathcal{A}(x)}{(\int_0^\infty \phi\tilde{R}_w^x + (1-\phi)R_w^x\mathrm{d}\mathcal{A}(x))^2}\right)$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}\tilde{\lambda}_\alpha} = -\phi\frac{\tilde{R}_b^\alpha}{(r+\tilde{\lambda}_\alpha)}\left(\frac{b(\alpha;\theta)}{\int_0^\infty \phi\tilde{R}_w^x + (1-\phi)R_w^x\mathrm{d}\mathcal{A}(\alpha)} + \frac{\int_0^\infty \phi b(x;\theta)\tilde{R}_b^x + (1-\phi)b(h_x;\theta)R_b^x\mathrm{d}\mathcal{A}(x)}{(\int_0^\infty \phi\tilde{R}_w^x + (1-\phi)R_w^x\mathrm{d}\mathcal{A}(x))^2}\right)$$

I assume that for every worker on the threshold have the same job finding rate noted $\lambda = \tilde{\lambda}$ and obviously the same affiliation noted h. Their duration in employment and unemployment are the same and are noted R_b and R_w. Since we are able to distinguish the reactions of our three groups of workers, the previous expression can be simplified:

$$\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = \frac{\partial\tau}{\partial\theta} + a_1\left(\frac{\partial h}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}h} + \varepsilon_{\lambda,b}\frac{\lambda}{b}\frac{\partial b}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}\lambda}\right) + a_2\frac{\mathrm{d}\tau}{\mathrm{d}h} + a_3\left(\varepsilon_{\lambda,b}\frac{\tilde{\lambda}}{b}\frac{\partial b}{\partial\theta}\frac{\mathrm{d}\tau}{\mathrm{d}\tilde{\lambda}}\right)$$

where $a_1 = (1-\phi)a(\theta-\Delta)$

$a_2 = (1-\phi)(\mathcal{A}(\theta) - \mathcal{A}(\theta-\Delta) - a(\theta-\Delta))$

$a_3 = \phi a(\theta)$

I note $V^{(i)}$ the present discounted value of workers of group i. The optimal θ is defined by the following formula:

$$
\begin{aligned}
&a_1 \frac{\partial V^{(1)}}{\partial \theta} + a_2 \frac{\partial V^{(2)}}{\partial \theta} + a_3 \frac{\partial V^{(3)}}{\partial \theta} + \int_0^\infty \left(\phi \frac{\partial \tilde{V}_\alpha}{\partial \tau} + (1-\phi) \frac{\partial V_\alpha}{\partial \tau} \right) \frac{\partial \tau}{\partial \theta} \mathrm{d}\mathcal{A}(\alpha) \\
&= -\int_0^\infty \left(\phi \frac{\partial \tilde{V}_\alpha}{\partial \tau} + (1-\phi) \frac{\partial V_\alpha}{\partial \tau} \right) \mathrm{d}\mathcal{A}(\alpha) \left(a_1 \left(\frac{\partial h}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}h} + \varepsilon_{\lambda,b} \frac{\lambda}{b} \frac{\partial b}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}\lambda} \right) + a_2 \frac{\mathrm{d}\tau}{\mathrm{d}h} + a_3 \left(\varepsilon_{\lambda,b} \frac{\tilde{\lambda}}{b} \frac{\partial b}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}\tilde{\lambda}} \right) \right)
\end{aligned}
$$

$$
\begin{aligned}
\rightarrow & a_1 \left(\frac{\partial V^{(1)}}{\partial \theta} + \frac{\partial V^{(1)}}{\partial \tau} \frac{\partial \tau}{\partial \theta} \right) + a_2 \left(\frac{\partial V^{(2)}}{\partial \theta} + \frac{\partial V^{(2)}}{\partial \tau} \frac{\partial \tau}{\partial \theta} \right) + a_3 \left(\frac{\partial V^{(3)}}{\partial \theta} + \frac{\partial V^{(3)}}{\partial \tau} \frac{\partial \tau}{\partial \theta} \right) \\
&+ \int_{\alpha \notin [\theta - \Delta; \theta]} \left(\phi \frac{\partial \tilde{V}_\alpha}{\partial \tau} + (1-\phi) \frac{\partial V_\alpha}{\partial \tau} \right) \frac{\partial \tau}{\partial \theta} \mathrm{d}\mathcal{A}(\alpha) + \int_{\alpha \in [\theta - \Delta; \theta[} \phi \frac{\partial \tilde{V}_\alpha}{\partial \tau} \frac{\partial \tau}{\partial \theta} \mathrm{d}\mathcal{A}(\alpha) \\
&= -\int_0^\infty \left(\phi \frac{\partial \tilde{V}_\alpha}{\partial \tau} + (1-\phi) \frac{\partial V_\alpha}{\partial \tau} \right) \mathrm{d}\mathcal{A}(\alpha) \left(a_1 \left(\frac{\partial h}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}h} + \varepsilon_{\lambda,b} \frac{\lambda}{b} \frac{\partial b}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}\lambda} \right) + a_2 \frac{\mathrm{d}\tau}{\mathrm{d}h} + a_3 \left(\varepsilon_{\lambda,b} \frac{\tilde{\lambda}}{b} \frac{\partial b}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}\tilde{\lambda}} \right) \right)
\end{aligned}
$$

$$
\rightarrow a_1 CS_1 + a_2 CS_2 + a_3 CS_3 + CS_{others} = -\frac{\partial W}{\partial \tau} (a_1 FE_1 + a_2 FE_2 + a_3 FE_3)
$$

The consumption smoothing loss of workers in the group $i \in \{1, 2\}$ is:

$$
\begin{aligned}
CS_i =& e^{-rh_i} \frac{\partial b(h_i;\theta)}{\partial \theta} \int_0^\infty e^{-(r+\lambda_i)t} u'(c_{t,i}) \mathrm{d}t \\
&- \left(u'(\underline{c}_i) \left(\frac{1 - e^{-rh_i}}{r} \right) + e^{-rh_i} \lambda_i \int_0^\infty e^{-(r+\lambda_i)t} \frac{u'(\overline{c}_{t,i})}{r} \mathrm{d}t \right) \left(\frac{\int_{\theta-\Delta}^{\theta} (1-\phi) \frac{\partial b(h_x;\theta)}{\partial \theta} R_b^x \mathrm{d}\mathcal{A}(x) + \phi a(\theta) \frac{\partial b(\theta;\theta)}{\partial \theta} \tilde{R}_b^\theta}{\int_0^\infty \phi \tilde{R}_w^x + (1-\phi) R_w^x \mathrm{d}\mathcal{A}(x)} \right)
\end{aligned}
$$

The consumption smoothing gain of unimpacted workers is:

$$
\begin{aligned}
CS_{others} = & -\int_{\alpha \notin [\theta - \Delta; \theta]} \phi \left(u'(\tilde{\underline{c}}_\alpha) \left(\frac{1 - e^{-r\alpha}}{r} \right) + e^{-r\alpha} \tilde{\lambda}_\alpha \int_0^\infty e^{-(r+\tilde{\lambda}_\alpha)t} \frac{u'(\tilde{\overline{c}}_{t,\alpha})}{r} \mathrm{d}t \right) \\
& + (1-\phi) \left(u'(\underline{c}_\alpha) \left(\frac{1 - e^{-rh_\alpha}}{r} \right) + e^{-rh_\alpha} \lambda_\alpha \int_0^\infty e^{-(r+\lambda_\alpha)t} \frac{u'(\overline{c}_{t,\alpha})}{r} \mathrm{d}t \right) \mathrm{d}\mathcal{A}(\alpha) \\
& \left(\frac{\int_{\theta-\Delta}^{\theta} (1-\phi) \frac{\partial b(h_x;\theta)}{\partial \theta} R_b^x \mathrm{d}\mathcal{A}(x) + \phi a(\theta) \frac{\partial b(\theta;\theta)}{\partial \theta} \tilde{R}_b^\theta}{\int_0^\infty \phi \tilde{R}_w^x + (1-\phi) R_w^x \mathrm{d}\mathcal{A}(x)} \right)
\end{aligned}
$$

The fiscal externality generated by each group i of impacted workers is:

$$
\begin{aligned}
FE_1 &= \left(\frac{\partial h}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}h} + \varepsilon_{\lambda,b} \frac{\lambda}{b} \frac{\partial b}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}\lambda} \right) \\
FE_2 &= \frac{\mathrm{d}\tau}{\mathrm{d}h} \\
FE_3 &= \varepsilon_{\lambda,b} \frac{\tilde{\lambda}}{b} \frac{\partial b}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}\tilde{\lambda}}
\end{aligned}
$$

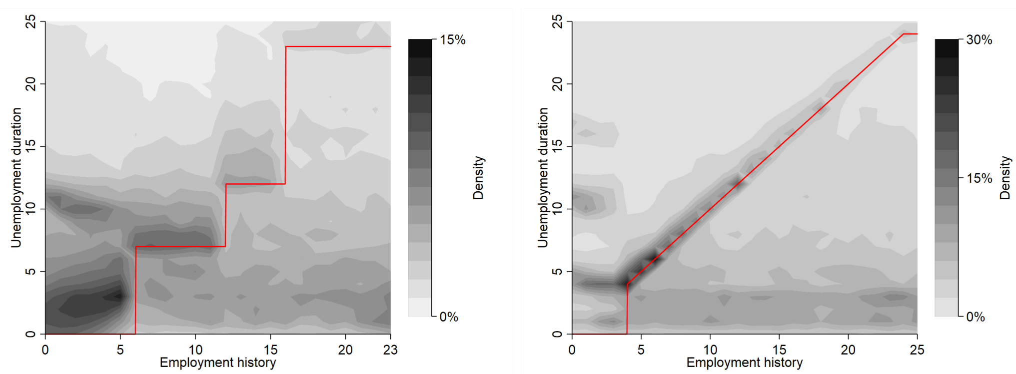
By assuming that every worker has a hand to mouth behavior (note $c_u = b$ and $c_e = w - \tau$) and by normalizing by $u'(c_e)$ the formula can be simplified as:

$$
\begin{aligned}
&(a_1 + a_2 + a_3)\left(\frac{\partial b(h;\theta)}{\partial \theta} R_b \frac{u'(c_u)}{u'(c_e)} - R_w \frac{\partial \tau}{\partial \theta}\right) \\
&- \left(\int_{\alpha \notin [\theta-\Delta;\theta]} \phi \tilde{R}_w^{\alpha} + (1-\phi) R_w^{\alpha} \mathrm{d}\mathcal{A}(\alpha) + \int_{\alpha \in [\theta-\Delta;\theta[} \phi \tilde{R}_w^{\alpha} \mathrm{d}\mathcal{A}(\alpha)\right) \frac{\partial \tau}{\partial \theta} \\
&= \int_0^{\infty} \phi \tilde{R}_w^{\alpha} + (1-\phi) R_w^{\alpha} \mathrm{d}\mathcal{A}(\alpha) \left(a_1 \left(\frac{\partial h}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}h} + \varepsilon_{\lambda,b} \frac{\lambda}{b} \frac{\partial b}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}\lambda}\right) + a_2 \frac{\mathrm{d}\tau}{\mathrm{d}h} + a_3 \left(\varepsilon_{\lambda,b} \frac{\tilde{\lambda}}{b} \frac{\partial b}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}\tilde{\lambda}}\right)\right) \\
&\rightarrow \left((1-\phi)\int_{\theta-\Delta}^{\theta} \mathrm{d}\mathcal{A}(\alpha) + \phi a(\theta)\right) \frac{\partial b(h;\theta)}{\partial \theta} R_b \left(\frac{u'(c_u) - u'(c_e)}{u'(c_e)}\right) \\
&= \int_0^{\infty} \phi \tilde{R}_w^{\alpha} + (1-\phi) R_w^{\alpha} \mathrm{d}\mathcal{A}(\alpha) \left(a_1 \left(\frac{\partial h}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}h} + \varepsilon_{\lambda,b} \frac{\lambda}{b} \frac{\partial b}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}\lambda}\right) + a_2 \frac{\mathrm{d}\tau}{\mathrm{d}h} + a_3 \left(\varepsilon_{\lambda,b} \frac{\tilde{\lambda}}{b} \frac{\partial b}{\partial \theta} \frac{\mathrm{d}\tau}{\mathrm{d}\tilde{\lambda}}\right)\right)
\end{aligned}
$$

B Additional material on the empirical part

B.1 Distributions of employment and unemployment durations

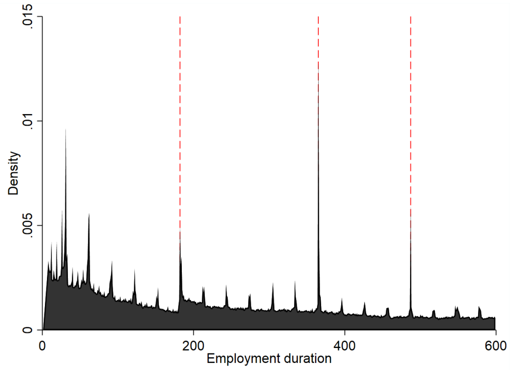
Figure 9: Distribution of d given h



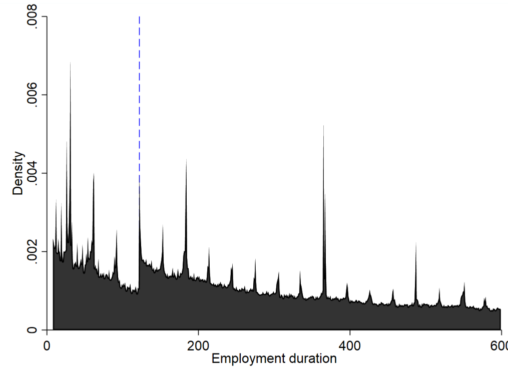
(a) Convention 2006 (b) Convention 2009

Note: This graph plots the density of each duration d for a given affiliation h. The unemployment and employment durations are expressed in months. The red line corresponds to the eligibility function, i.e the potential benefit duration for each employment history.

Figure 10: Distribution f of h



(a) Convention 2006



(b) Convention 2009

Note: Graph a plots the distribution of employment history h under the 2006 convention. The three red dash lines correspond respectively to 6, 12 and 16 months, the three discontinuities in the eligibility function under the 2006 convention. Graph b plots the distribution of employment history h under the 2009 convention. The blue dash lines corresponds to 4 months, the eligibility criterion under the 2009 convention. The unemployment durations are expressed in days.

B.2 Bunching: robustness

	(1)	(2)	(3)	(4)	(5)	(6)
Degree	1	3	7	5	5	5
Counterfactual	2006	2006	2006	2004	2006	2006
Excess mass segment Δ_+	3	3	3	3	5	7
Missing mass segment Δ	5***	6***	7***	5***	7***	10***
	(.4688)	(.3775)	(.7160)	(.6000)	(.6211)	(.6050)
Frictions ϕ	.73***	.76***	.81***	.77***	.77***	.77***
	(.0107)	(.0123)	(.0134)	(.0160)	(.0123)	(.0096)

Standard errors in parentheses (bootstrap 500)

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Figure 11: Robustness for bunching at 4 months

Table 11 and 12 display the results of different robustness checks. I successively test the effect of change in the polynomial degree $\overline{p}$, the convention used as counterfactual for the cyclical separations within the notche range or the convention used as sample of interest and the choice of the excess segment Δ^+. The results seem to be fully robust to a change in polynomial and counterfactual. Increasing the the segment Δ^+ has a marginal impact on the results but it remains consistent.

	(1)	(2)	(3)	(4)	(5)	(6)
Degree	1	3	7	5	5	5
Convention	2006	2006	2006	2004	2006	2006
Excess mass segment Δ_+	5	5	5	5	7	9
Missing mass segment Δ	11***	11***	11***	11***	13***	15***
	(.6006)	(.6854)	(.8100)	(.7833)	(.9344)	(1.450)
Frictions ϕ	.82***	.82***	.84***	.77***	.84***	.83***
	(.0087)	(.0101)	(.0097)	(0.0143)	(.0095)	(.0111)

Standard errors in parentheses (bootstrap 500)

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

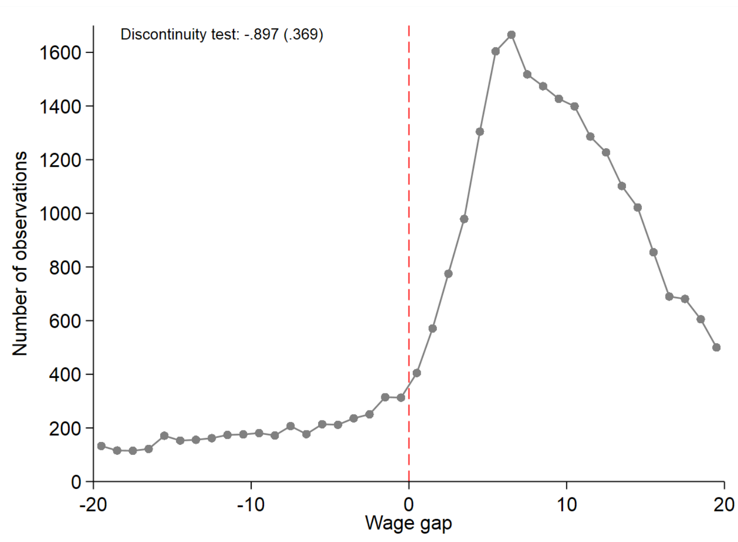
Figure 12: Robustness for bunching at 6 months

B.3 RKD: robustness

B.3.1 Validity of assumptions

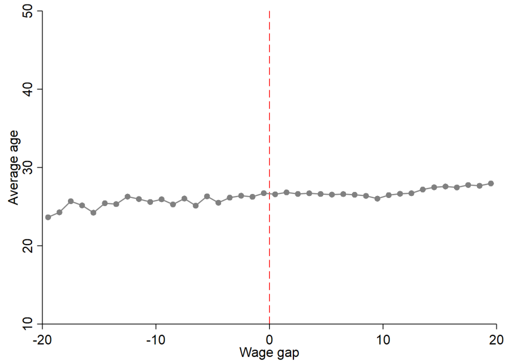
Firstly, the density of the reference wage must be smooth, i.e continuously differentiable, at the kink. Figure 13 displays the density of the assignment variable, i.e the reference wage around the kink. The density seems to be smooth at the kink. A McCrary test confirms these observations.

Figure 13: Number of observations

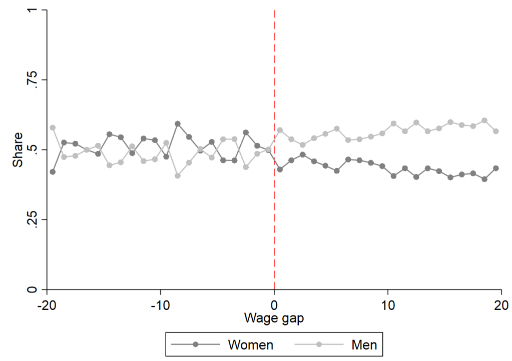


Secondly, individuals have to be similar on both sides of the kink. The individual's characteristic must be smooth at the kink.

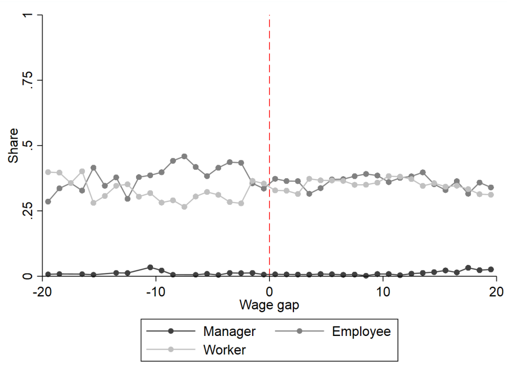
Figure 14: Smooth characteristics



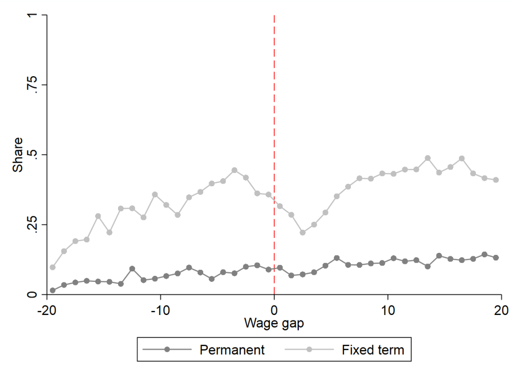
(a) Age



(b) Sex



(c) Social class



(d) Contract

B.3.2 Alternative specifications

I test the sensibility of my results to the degree of the polynomial, the considered kink and the bandwidth choice. I also use Tobit model to show that censoring does not have any effect on my results.

The Tobit model yields a LATE of 4.34 (1.007) and an elasticity of .463 (.107). Again, it confirms my results.

	(1)	(2)	(3)	(4)	(5)
Kink	k_1	k_1	k_1	k_1	k_2
Degree	2	3	1	1	1
Bandwidth	20	20	25	15	20
LATE	10.67^{**}	25.02	4.04^{***}	6.04^{***}	4.79^{***}
	(4.90)	(15.56)	(.796)	(1.575)	(2.40)
ε_b	1.14^{**}	2.67	$.43^{***}$	$.64^{***}$	$.88^{***}$
	(.522)	(1.667)	(.085)	(.168)	(.439)
N	23243	23243	25640	19225	20645

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 5: RKD alternative specifications

Note: unemployment duration is defined as d. A t test (p=.19) confirms that the results are consistent when I consider k_2.